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Cognitive Constraints on Ordering Operations: The Case of Geometric Analogies

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SUMMARY

Many tasks (e.g., solving algebraic equations and running errands) require the execution of several component processes in an unconstrained order. The research reported here uses the geometric analogy task as a paradigm case for studying the ordering of component processes in this type of task. In solving geometric analogies by applying mental transformations such as *rotate, change size,* and *add a part,* the order of performing the transformations is unconstrained and does not in principle affect solution accuracy. Nevertheless, solvers may bring cognitive constraints with them to the analogy task that influence the ordering of the transformations. First, we demonstrate that solvers have a preferred order for performing mental transformations during analogy solution. We then investigate three classes of explanations for the preferred order, one based on general information processing considerations, another based on task-specific considerations, and a third based on individual differences in analogy ability. In the first and third experiments, college students solved geometric analogies requiring two or three transformations and indicated the order in which they performed the transformations. There was close agreement on nearly the same order for both types of analogies. In the second experiment, subjects were directed to perform pairs of transformations in the preferred or unpreferred order. Both speed and accuracy were greater for the preferred orders, thus validating subjects' reported orders.

Ability differences were observed for only the more difficult three-transformation problems: High- and middle-ability subjects agreed on an overall performance order, but the highs were more consistent in their use of this order. Low-ability subjects did not consistently order the transformations for these difficult problems. The general information processing factor examined was working-memory load. A number of task factors have been shown to affect working-memory load during the solution of inductive reasoning problems. Of these, we chose to examine process difficulty. Because analogies are solved in working memory, performing more difficult transformations earlier may reduce working-memory load and facilitate problem solution. However, the observed performance order was not correlated with transformation difficulty. The first task-specific factor considered was that some transformations may be identified earlier, possibly because of perceptual salience, and that the performance order follows the identification order. However, the order of transformation identification also did not account for the order of transformation application in either experiment.

Solving geometric analogies is an imagery task in which the solution is constructed mentally by performing the inferred transformations on the given figure. Performance on geometric analogy tests is moderately to highly correlated with a variety of spatial visualization tests that involve image generation, transformation, and retention. The second task-specific factor considered was that the order of transformation application would resemble the order of using the corresponding information in an analogous physical task, the planning and execution of a drawing. Thus, in the fourth experiment, subjects were asked to indicate the order in which they would need specific types of information, each corresponding to a transformation, in order to draw a simple picture. This drawing order paralleled the order of performing mental transformations during analogy solution. Perceptual processing during object identification is also shown to proceed in a similar order. These results suggest that in solving geometric analogies, subjects tap into procedures and constraints common to the domains of imagery, drawing, and object identification.

In sum, we have shown that a logically unconstrained ordering of component processes does not necessarily imply that the observed ordering will be random. Rather, cognitive constraints imported from more familiar domains were shown to impose substantial agreement on the ordering of mental transformations during geometric analogy solution. We suspect that this type of finding is likely to obtain for other tasks that have theoretically unconstrained orders of component operations.

During the Rubik's Cube craze of the early 1980s, three books that described solutions simultaneously captured the first, second, and fourth places on the *New York Times* paperback bestseller list (Tierney, 1986). In the execution of the 52 steps on the shortest solution path, the cube appears disorganized until the very end. Two longer solution paths (as many as 120 moves in one case) yield a series of intermediate products that are recognizably closer to the final solution. Why did solvers generally prefer the longer methods? The longer solutions are chunked into meaningful units, so the extra steps are compensated for by a reduction in working-memory load. In contrast to the richness of Rubik's Cube, many tasks have essentially only a single solution path with a prescribed sequence of moves (discounting backtracking; e.g., Missionaries and Cannibals, puzzles in which two or more intertwined pieces of wire must be disentangled). For these tasks, solvers must perform the requisite steps in the prescribed order if they are to succeed.

More interesting are an intermediate set of tasks, from solving algebraic equations to running errands, that require the performance of a single set of component operations, but for which the order of executing the operations is optional. In these tasks, as in Rubik's Cube, solvers have a choice of solution paths. Thus, here also there may be cognitive reasons, such as reducing working-memory load, for selecting one sequence over another. Solving geometric analogies is a familiar and convenient example of such a task (see the top row of Figure 2 for a sample analogy). Several simple operations or transformations on geometric figures, such as *move* and *add half*, are inferred from the example terms (A and B) and then performed on the test term (C) to generate the solution (D). The order of performing the transformations is optional; that is, it does not matter whether one moves the figure first and then adds the other half to it or adds the other half first and then moves the figure, because the correct solution can be obtained in either case. Unlike the case of algebraic equations, there has been no formal instruction in geometric analogy solution techniques, instruction that biases the order of performing operations. And unlike running errands, there are no extraneous motivational factors (shall I go to the bakery first to get encouragement or last as a reward?) to bias the order.

Geometric analogies have been studied by several researchers interested in intelligence and analogical reasoning (Bethell-Fox, Lohman, & Snow, 1984; Mulholland, Pellegrino, & Glaser, 1980; Spearman, 1923; Sternberg, 1977; Whitely & Schneider, 1981). Using solution time as a dependent measure, Sternberg (1977) has isolated several solution stages, including (see also Spearman, 1923, for a discussion of the first, second, and fourth of these components) (a) *encoding* the terms of the analogy, (b) *inferring* the rules relating the two example terms (A and B), (c) *mapping* the rules relating the A and C terms, and (d) *applying* the rules identified at the inference stage: that is, performing the transformations on the C term to yield the answer, the D term. These global components of analogy solution have a prescribed order (e.g., a particular transformation must be inferred before it can be applied/performed), but the order of performing the individual transformations on the test term to produce an answer is unconstrained. Although problem solvers conceivably could perform multiple transformations in parallel, the work of Sternberg (1977) and Mulholland et al. (1980) suggests that transformations are performed serially during the application stage of solution. The optionality of transformation ordering then becomes important. In particular, one performance order may be more cognitively compelling than another. The order in which multiple transformations are performed on a single geometric figure may also have consequences for accuracy and solution time.

The research presented in this article will demonstrate that problem solvers have a preferred order for performing transformations during geometric analogy solution, despite the fact that the order is entirely optional. Three classes of explanations to account for this consistency will be considered. The first is based on general constraints of information processing, the second on task-specific considerations, and the third on individual differences. These classes of explanations are not mutually exclusive; thus, the task at hand is one of seeking evidence for each, rather than of deciding among them. For each class, the general claims will be described first, followed by the specific predictions tested experimentally.

General Information Processing Constraints: Working Memory

The solutions to many problems require the mental assembly and transformation of several different elements or processes. The greater the number of elements and/or processes, the greater the load on working memory. In general, there is a trade-

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off between the amount of information that can be stored in working memory and the amount of processing that can be done (e.g., Case, 1978; Daneman & Carpenter, 1980; Newell & Simon, 1972). Working-memory load has been implicated in the performance of inductive reasoning tasks in general, and of analogies in particular (see Bethell-Fox et al., 1984, Mulholland et al., 1980, Sternberg, 1977, and Sternberg & Rifkin, 1979, for geometric and related analogies; Holzman, Pellegrino, & Glaser, 1982, 1983, for number analogies and number series; and Kotovsky & Simon, 1973, for letter series problems). These studies have shown that increasing the number and/or difficulty of the requisite processes increases demands on working memory and decreases solution success.

Working memory has proved to be an important theoretical construct in other domains as well, for example, reading (e.g., Daneman & Carpenter, 1980) and mental arithmetic (e.g., Hitch, 1978a, 1978b). Hitch's work on mental arithmetic is particularly relevant here because he has shown that intermediate results not immediately produced as responses are an added burden to working memory. Furthermore, he has suggested "that one feature of 'good' cognitive strategies is that they will minimize any deleterious effects of short-term forgetting on performance. It seems likely that this generalization will apply most clearly in tasks that are serial in nature, with a degree of option about the sequence of stages" (Hitch, 1978a, p. 337). The application of transformations in the solution of geometric analogies is a good example of such a situation.

One simple way geometric analogy solvers might reduce working-memory load is by performing transformations in decreasing order of difficulty. When two or more transformations are applied in series to a single geometric figure, the first transformation is applied to the externally presented (and internally represented) C term, but subsequent transformations are applied to the mentally represented products of the earlier transformations. More difficult transformations may take more time and/or more effort and therefore might benefit more from the externally represented figure than easier transformations. Alternatively, easier transformations might be more resistant to disruption than harder transformations when performed on a figure that is represented solely internally. Thus, for the general hypothesis that problem solvers order operations to reduce working-memory load, we will examine whether transformations are performed in order of decreasing difficulty. This is obviously not the only way of instantiating the working-memory hypothesis. However, it seemed that it would be a sensible and fairly easy strategy for subjects to adopt.

Task-Specific, Content-Dependent Considerations: Perceptual Processing

Persistent failures to find transfer between problems with similar solutions (e.g., Gick & Holyoak, 1983) and the lack of transparency of problem isomorphs (e.g., Hayes & Simon, 1977; Simon & Hayes, 1976) lend support to the idea that many important problem-solving strategies are content bound and task specific. In geometric analogy solution, perceptual factors may be important in determining the order in which transformations are initially inferred; some transformations may be more salient than others, and these may be detected earlier. One simple task-specific strategy would be to apply the transformations in the order in which they were identified. This strategy has the advantage of saving the step of deciding the transformation order separately for each problem.

A more complex task-specific constraint is suggested by the observation that constructing a geometric analogy solution typically entails constructing a mental image (Bethell-Fox et al., 1984). Many of the transformations used in geometric analogies have been studied separately in imagery tasks, in particular, mental rotation (Cooper & Shepard, 1973; Shepard & Cooper, 1982), mental size scaling (Besner & Coltheart, 1976; Bundeson & Larsen, 1975), and mental scanning (Kosslyn, Ball, & Reiser, 1978). Although theories of imagery posit that several transformations may be performed in sequence during image construction (Kosslyn, 1980; Kosslyn, Brunn, Cave, & Wallach, 1984; Shepard, 1984), imagery studies typically have examined transformations only singly, and the theories are moot with regard to the ordering of these transformations. Piaget and Inhelder (1956, 1971) have observed that construction of a mental image may bear resemblance to construction of a pictorial image. Thus, the order of performing transformations to mentally construct a geometric analogy solution may mirror the order in which different types of information about an object are needed to efficiently plan and execute a drawing of that object.

Individual Differences Considerations

Pellegrino and Glaser (1980) have suggested that individual differences may be expressed within the various components of analogy solution. With respect to transformation application, there are at least two possibilities. First, high- and low-ability subjects may adopt different performance orders that are determined by different constraints. For example, the order used by high-ability subjects might be determined by general constraints, such as working-memory load, whereas the order used by low-ability subjects might be determined by task-specific constraints, such as perceptual factors (see also Hunt, 1974). Consistent with this notion, Schiano, Cooper, and Glaser (1984) found that high-ability high school students tended to sort geometric analogies on the basis of the transformational relations among the figures, but that low-ability subjects generally sorted on the basis of the surface similarities of the figures. A second possibility is that high- and low-ability subjects might adopt essentially the same transformation order but differ in the consistency with which they follow that order. It is not unusual to find consistency of strategy use covarying with ability (see, e.g., Campione, Brown, & Ferrara, 1982; Sternberg, 1977).

Experimental Program

Four experiments will be reported, directed at both establishing and understanding the order of performing mental transformations in the solution of geometric analogies. In the first experiment, order was studied in two-transformation analogies. Throughout the research, it is assumed that transformations are performed in sequence or cascades, not in parallel. Although this assumption has empirical support (Mulholland et al., 1980;

Sternberg, 1977), it does restrict the conclusions to problems for which the operations are not begun simultaneously. Determination of the order of transformation application relies, for the most part, on subjects' reports. The validity of these reports was checked in the second experiment, in which subjects were directed to solve analogies by performing the transformations in the preferred or unpreferred order, and speed and accuracy of solution were recorded. In the third experiment, the robustness of the performance order was tested using three-transformation problems. Finally, in the fourth experiment, order was examined in the construction of pictures.

Experiment 1

The first experiment was designed to determine whether there is a preferred order for performing transformations in the solution of geometric analogies, as well as to collect evidence bearing on the three classes of explanations. A consistent and transitive order of transformation application would be a strong finding because the task in no way constrains the order; problem solvers are free to perform the transformations in any order they choose because the order of application does not affect the identity of the final outcome. The transformation difficulty instantiation of the working-memory hypothesis was assessed by comparing the performance order to the order of the transformations in terms of difficulty. The hypothesis that transformations are applied in the order in which they are initially identified was tested by recording identification order. Finally, ability measures were collected to examine performance order as a function of ability.

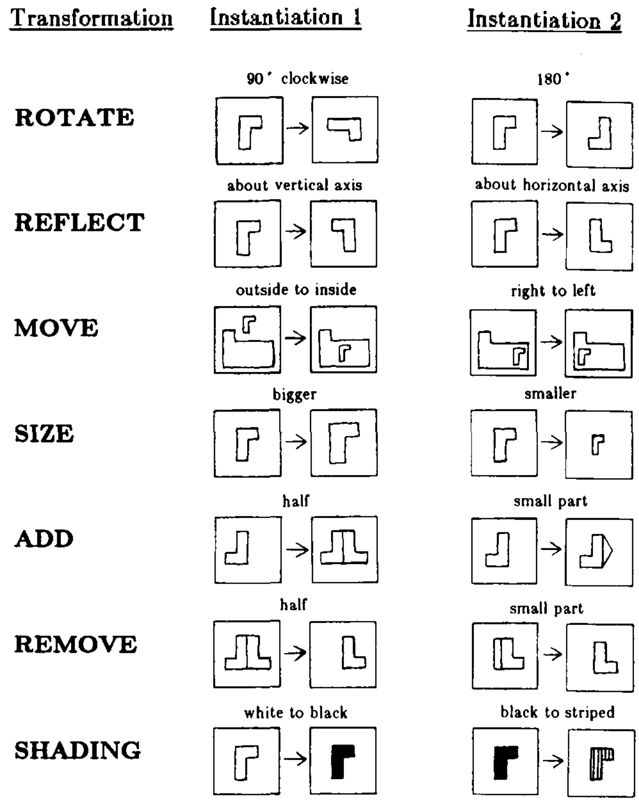

Figure 1. Examples of the two instantiations of each of the seven basic transformations.

Method

Subjects

The subjects were 98 Stanford University undergraduates who participated in partial fulfillment of course requirements. The data from 2 of these subjects were excluded from the analyses because those subjects failed to follow the instructions for the experimental task.

Stimuli

All of the analogies required two transformations to be performed on a single geometric figure. There were 21 problem types, which were chosen by constructing all possible pairs of the following seven basic transformations: *rotate, reflect, move, size, add, remove,* and *shading.* These transformations represent a variety of analog and discrete transformations that objects commonly undergo and that have previously been studied as component processes in cognitive tasks (e.g., Besner & Coltheart, 1976; Bundeson & Larsen, 1975; Cooper & Shepard, 1973; Shepard, 1975), and that are commonly found on psychometric analogy tests. Two different instantiations of each of these basic transformations were used in the analogies, as illustrated in Figure 1. Four analogies were constructed for each problem type by factorially combining the two instantiations of each transformation. For example, the four *rotate/size* problems were *rotate 90°/bigger, rotate 90°/smaller, rotate 180°/bigger,* and *rotate 180°/smaller.* Thus, there were 84 problems in all.

To construct analogies for which subjects would identify the intended transformations, there had to be two exceptions to the factorial combination rule. First, because a 180° rotation combined with a reflection looks identical to a single reflection about the axis perpendicular to the original reflection, the two rotate transformations used for the *rotate/reflect* problems were rotate 90° clockwise and rotate 90° counterclockwise. Second, because of difficulty in constructing an *add-half/remove-half* problem, an additional *add-part/remove-part* problem was used instead. To ensure generality across figures, many different geometric figures were used to construct the analogies.

Design

There were two conditions in this experiment: a *solution* condition ($N = 48$; 23 female and 25 male subjects) and an *identification* condition ($N = 48$; 20 female and 28 male subjects). In the solution condition, subjects saw the first three (A, B, and C) terms of an analogy and solved the problem by mentally constructing the fourth (D) term. The correct D term was presented on the other side of the page as one of five multiple-choice alternatives. Thus, subjects had to use the imaginal constructive matching solution strategy described by Bethell-Fox et al. (1984). The position of the correct answer was randomly chosen for each problem type, with the constraint that each position contained the correct answer approximately equally often. The position of the correct answer was held constant for all four analogies representing a given problem type. The four distractors for each analogy were constructed according to the following rules: (a) Transformation X alone applied to the third figure, (b) Transformation Y alone applied to the third figure, (c) Transformation X performed correctly and Transformation Y performed incorrectly, and (d) Transformation X performed incorrectly and Transformation Y performed correctly. A sample *move/add-half* problem appears in Figure 2. Subjects in the identification condition also saw the

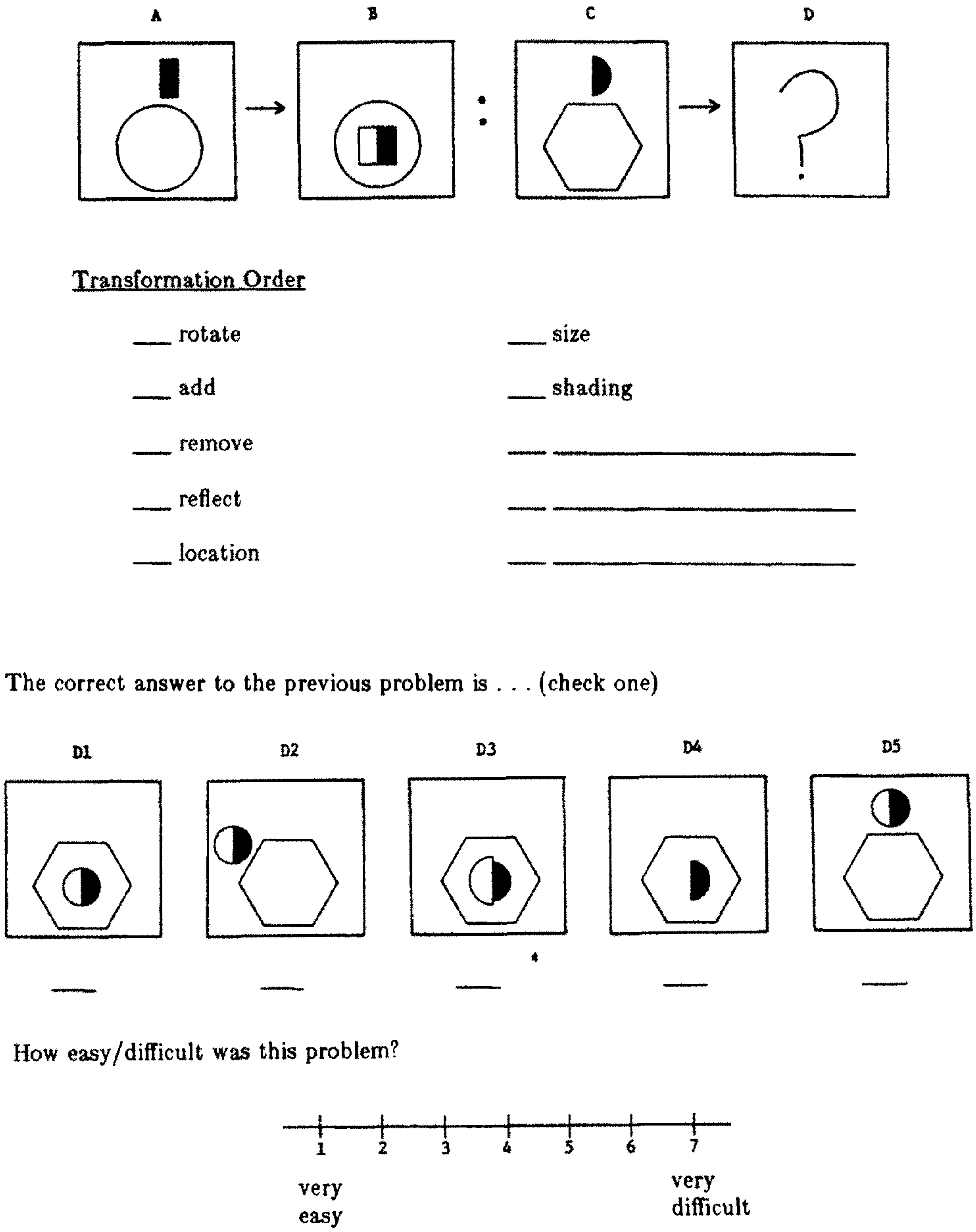

Figure 2. Example of a *move/add* problem from the solution condition, including the five alternative answers.

first three terms of the analogies (see the top part of Figure 2) but simply had to identify the transformations that were used to change the A figure into the B figure.

Subjects completed 1 problem from each problem type for a total of 21 problems. The problems were randomly chosen from among the 4 problems for each problem type such that each problem was completed by 12 subjects (hence the 48 subjects in each condition). Six different orders for presenting the problems were used, with 8 subjects receiving each order. The problem orders were random given the constraint that none of the seven basic transformations could appear in two consecutive problems.

Procedure

Subjects in both conditions began the experiment with a 24-item multiple-choice geometric analogy test, on which they were given 7 min to work. This test served two purposes. First, it gave subjects some practice solving geometric analogies before they had to solve the experimental problems, and, second, it served as a measure of analogical reasoning ability. The problems on this test were similar to those found on published psychometric tests. (A copy of the test may be obtained by writing to Laura R. Novick.)

After completing the analogy test, subjects were given detailed oral instructions that described the types of transformations they would encounter and the steps they were to follow in solving the problems. The instructions for the two conditions were identical until the experimenter began describing the specific task. The solution condition subjects' task was as follows: First, they were to identify the transformations used to change the A figure into the B figure. Second, they were to mentally apply those same transformations to the C figure to derive the correct answer. Third, they were to number the transformations in the order in which they had mentally performed them during the application stage. The seven basic transformations used in the experiment were printed below each analogy (see Figure 2). In addition, there were three blank lines at the end of the transformation list in case subjects used other transformations (a few subjects on a few problems actually used novel transformations; other subjects wrote descriptions of transformations that were functionally identical to the verbal labels we provided). Although subjects were not told that they had to perform the transformations sequentially, the instructions did imply that they do so. Thus, our results are necessarily restricted to situations in which transformations are performed sequentially (or at least to those in which one transformation is started before the other); although the data of Sternberg (1977) and Mulholland et al. (1980) suggest that this may typically be the case. To the extent that subjects performed transformations simultaneously, there should be little, if any, consistency in the ordering of transformations across subjects.

After numbering the transformations, subjects turned the page over and marked the answer they had constructed mentally. Subjects were instructed not to look back. Thus, if the answer they had constructed was not among the alternatives, they were to indicate their best guess. Finally, subjects rated the difficulty of the problem on a 7-point scale (1 = *very easy,* 7 = *very difficult*). This procedure was followed for all 21 problems in the booklet.

The identification condition subjects had only a single page per problem, which was identical to the first page given to the solution condition subjects. Their task was to identify the transformations used to change the A figure into the B figure and number those transformations in the order in which they noticed them. The identification condition subjects did not actually solve the analogies and could ignore the C figure printed on the problem page. However, it was important that the identification condition subjects see the same display as the solution condition subjects because the identification order was hypothesized to be determined by transformation salience, and the presence or absence of the C figure could affect the relative salience of the two transformations.

Results and Discussion

Solution Condition: The Order in Which Transformations Are Performed

The transformation ordering data were analyzed by using a nonparametric scaling procedure based on paired-comparison data (Carroll & Chang, 1964; Chang & Carroll, 1968). The data consisted of a 7 × 7 paired-comparison matrix for each subject, with the cell entries indicating whether the transformation in Row *i* was performed before or after the transformation in Column *j*. A cell entry was coded as missing if the subject marked an incorrect answer alternative or failed to number the correct transformations (inclusion of the former problems does not change the transformation ordering results). Because one of the four analogies from the *rotate/add* problem type was inadvertently misdrawn, the 12 subjects who received that problem were coded as having missing data for this pair of transformations.

Finding the transformation order that best represents the paired-comparison data is a two-step procedure. In the first step, each subject's paired-comparison data matrix is processed separately. The program determines for each subject the number of times each transformation was performed before the other transformations. From this information, it determines the best overall ordering of the transformations for each subject and assigns each transformation a score that reflects the extent to which the subject performed that transformation first. Thus, the output from this step is a number-of-subjects by number-of-transformations matrix, with each row representing the transformation scores for a particular subject.

In the second step, this matrix is factored to yield two geometric configurations, one of transformations and one of subjects. Although the program is capable of representing objects in a multidimensional space, we specified a one-dimensional solution because it is most appropriate in the present context. In one dimension, the configuration of transformations represents the linear (metric) order that best captures the order in which subjects performed the transformations. The transformations are plotted as coordinates on a line to represent the metric properties of the solution. The configuration of subjects specifies, for each subject, the end of the linear transformation order that best corresponds to the beginning of that subject's order. To the extent that the transformations are performed in a consistent order across subjects, the subjects should cluster at one end of the transformation order.

The resulting transformation order is presented in Figure 3a. *Move,* on the far left, is performed first, *rotate* and *reflect* are next, then *remove,* then *size,* and, finally, *add* and *shading.* There are several ways to assess how well this solution fits the data. First, this order is the best representation of the transformation scores for 47 of the 48 subjects; the remaining subject tended to perform the transformations in exactly the opposite order (i.e., *shading* first and *move* last). The average Pearson correlation between the transformation scores of all pairs of subjects is .45. The average Spearman rank-order correlation is .44. This corresponds to a Kendall's coefficient of concordance (ω) of .45, $\chi^2(6, N = 48) = 129.12$, $p < .001$. As hypothesized, subjects agree on the order in which they perform mental transformations. If the subject whose order is the opposite of everyone else's is removed, the average rank-order correlation increases to .48, $\omega = .49$, $\chi^2(6, N = 47) = 139.25$, $p < .001$. These correlations are quite impressive considering that there are 5040 (7!) different ways subjects could order the transformations.

In the scaling analysis just reported, the data for each transformation pair were collapsed across the four problems representing that pair. To determine whether this was justified, we did follow-up chi-square tests for each transformation pair that explicitly examined performance order as a function of how the transformations were instantiated. For only 2 of the 21 problem

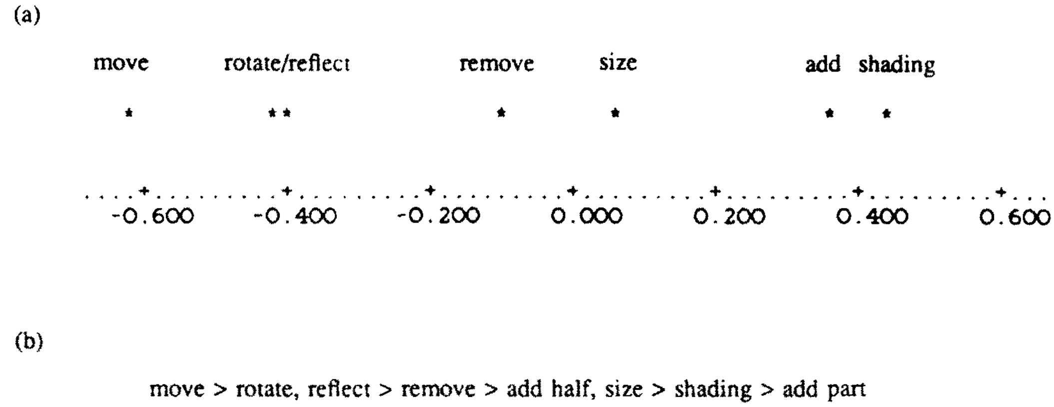

Figure 3. (a) Order (from left to right) in which mental transformations are performed in the solution of two-transformation geometric analogies as determined by the scaling analysis and (b) the revised performance order based on follow-up analyses.

types did the ordering of the transformations depend on the particular instantiations. For both the *size/add* and *add/shading* problem types, the order depended on the type of *add* transformation, $\chi^2(1, N = 39) = 14.04$, $p < .001$, and $\chi^2(1, N = 38) = 10.64$, $p < .01$, respectively. These dependencies can be incorporated into the Figure 3a order by putting *add half* next to *size* (those two transformations were not consistently ordered, $p > .11$ by a binomial test) and by putting *add part* after *shading* (and, of course, removing the original *add* transformation). The resulting priority order is shown in Figure 3b. Note that this transformation order is transitive (i.e., there is no triple of transformations for which subjects perform X before Y, Y before Z, and Z before X). This is a particularly strong finding given that no attempt was made to standardize the figures being transformed in the various problem types.

General Constraints on Problem Solving: Transformation Difficulty as a Predictor of the Performance Order

The particular working-memory hypothesis tested was that transformations are performed in decreasing order of difficulty. To assess the difficulty of each of the eight transformations (considering *add half* and *add part* separately), we computed the average number of errors across all problem types involving that transformation. The following order results: *rotate* (7.6 errors out of 48 = 15.8%) > *size* (13.1% errors) > *reflect* (11.0%) > *shading* (10.6%) > *add half* (10.4%) > *move* (7.7%) > *add part* (6.3%) > *remove* (4.2%). A comparison of this order and Figure 3b shows clearly that the relative difficulties of performing single transformations cannot account for the order in which multiple transformations are performed. The Spearman rank-order correlation between the two orders is .20, which is not significantly different from zero. *Move* is a particularly blatant offender. It results in relatively few errors, yet it is most likely to be applied first. *Remove* also presents a problem because it is in the middle of the performance order but results in the fewest number of errors. Thus, there is no evidence for the transformation difficulty version of the working-memory hypothesis.

Task-Specific Constraints: Identification Order as a Predictor of the Performance Order

The identification condition was designed to test the simple perceptual explanation that subjects perform the transformations in the order in which they identify them. This explanation carries with it the implicit assumption that the identification order is influenced primarily by perceptual factors such as transformation salience, although this assumption was not tested. We believe that transformation identification is determined to a large extent by transformation salience, but even if this is not the case, it could still be a reasonable strategy to use the identification order at the application stage of solution. The best-fitting identification order from the scaling program represents the data for only 32 of the 48 subjects (the reverse order is best for the remaining 16 subjects). The average intersubject correlation (Pearson or Spearman) is only .07, which represents a very small, but significant, degree of intersubject agreement, $\omega = .09$, $\chi^2(6, N = 48) = 26.59$, $p < .001$. Although the identification and performance orders are significantly correlated ($r_s = .87$, $p < .05$), suggesting that the order in which transformations are performed might depend on the order in which they are initially identified, this is not likely to be true for other reasons.

First, if subjects performed the transformations in the order in which they identified them, the agreement among subjects in the solution condition should be about the same as in the identification condition. However, it is six to seven times larger. Second, consistent with the hypothesis that identification order is affected by perceptual salience, some of the disagreement among subjects is due to the fact that although all subjects received the same pairs of transformations, the particular instantiations of the transformations differed. For example, *move* tended to be identified first in problems involving a small figure moving from outside a larger figure down inside that figure, but not in problems involving the movement of a small figure from the right side to the left side of the larger figure it is in. This pattern of results obtains when *move* is combined with *size*, *add*, or *shading*. Dependency of the identification order on the particular instantiations of the transformations was also observed for the *rotate/size* problem type. These dependencies were not found in the solution condition data. Thus, to the extent that there is any systematicity at all in the identification order, it tends to resemble the performance order; but some other mechanism must be accounting for the very high degree of consistency for the performance order.

Individual Differences Constraints: Ability as a Predictor of the Performance Order

Descriptive analysis of analogy measures. In addition to the 98 subjects in the experiment reported here, 30 subjects from an unreported rating task and 49 pilot subjects from the same population also completed the timed analogy test prior to the experimental task. The responses from all 177 subjects were included in the item analyses for the test so that more stable correlations could be obtained. Although all 24 items on the test correlate positively with overall performance, for the last item, total score correlates almost as highly with one of the distractors ($r = .124$) as with the correct answer ($r = .176$). Therefore, the responses from this item were removed and the analyses were redone on the 23-item test. The correlations between overall performance on the revised test and individual item scores range from .13 to .54, with a mean of .35. The Cronbach's alpha reliability of the 23-item test is .68 for these subjects. The item difficulties (i.e., the proportion of subjects solving each item correctly) range from .18 to .99, with a mean of .82 and a median of .90. Performance on the analogy test does not differ across conditions, $t(94) = 0.49$, $p > .62$, with an overall mean of 18.7 out of 23 (the scores range from 9 to 23).

We also computed number correct scores for the experimental tasks. In the identification condition, a problem was counted as correct if the transformations that were identified would correctly change the A figure into the B figure. The mean number correct was 17.3 out of 21, with a range of 12 to 21. Solution condition subjects had to mark the appropriate transformations and choose the correct D term. The mean for this condition was 18.8, with a range of 14 to 21. Both identifying and applying transformations are significant sources of individual differences in analogy ability (see also Sternberg, 1977), as the scores on the experimental task for both conditions correlate with performance on the timed analogy test ($r = .43$, $p < .003$ for the solution condition and $r = .35$, $p < .02$ for the identification condition). Although the .43 correlation for the solution condition may seem low given that both tasks involve solving geometric analogies, because the experimental analogies were untimed, there is a restricted range of scores for that task (particularly above the mean because subjects averaged 90% correct). This would tend to attenuate the correlation between the two measures.

Ability hypotheses. This class of hypotheses states that the order in which transformations are performed covaries with ability. As indicated earlier, this general hypothesis can be instantiated in at least two ways. First, high-ability subjects might perform the transformations in a different order than do low-ability subjects. Alternatively, highs and lows generally might perform the transformations in a similar order, but highs might be more consistent in their ordering than lows. To examine these hypotheses, we ran the scaling program on the data from high- and lower ability subjects separately. The 15 subjects in approximately the top third (31%; scores of 21–23) of the analogy test distribution were considered high in analogy ability. The 13 subjects in approximately the bottom third (27%; scores of 12–17) were lower in analogy ability.

The solutions for the two ability groups yield very similar application orders ($r_s = .93$, $p < .02$), which in turn are almost identical to the overall order shown in Figure 3a. The high-ability order is the same as the Figure 3a order except that *add* and *shading* are reversed. The lower ability order is the same except that *rotate* and *reflect* are reversed. The two groups also do not differ in terms of consistency. The average rank-order correlation of the transformation scores among the high-ability subjects is .53, $\omega = .56$, $\chi^2(6, N = 15) = 50.83$, $p < .001$. For the lower ability subjects, the average rank-order correlation is .49, $\omega = .53$, $\chi^2(6, N = 13) = 41.33$, $p < .001$. Contrary to the ability hypothesis, both high- and lower ability subjects are very consistent in their ordering of the transformations.

Thus, in the ability range represented, individual differences do not appear to be related either to the order in which transformations are performed or to the consistency with which the overall preferred transformation order is followed. However, ability differences may appear with more difficult problems and/or a wider range of ability, a possibility examined in Experiment 3.

Experiment 2

Problem solvers clearly have a preferred order for applying mental transformations in the solution of geometric analogies, and this order is consistent across subjects. Does using this order, as opposed to the opposite order, have consequences for task performance? In this experiment, subjects were told the order in which to perform the transformations for each problem, and solution times were recorded. If the order of transformation application reported by subjects in the first experiment is accurate and is determined by cognitive constraints, performance should be faster and better when subjects apply transformations in the preferred order. Thus, this experiment provides important corroboration of the self-report data from Experiment 1. If subjects' reports about the order in which they performed the transformations are not accurate or if the consistency of the reported order reflects some other factor unrelated to the actual order or to any cognitive constraints on that order, then no differences attributable to transformation ordering should be observed in this experiment.

Method

Subjects

The subjects were 32 Stanford University undergraduates (14 female and 18 male subjects) who participated in partial fulfillment of course requirements.

Stimuli

The stimuli were the 84 analogies used in Experiment 1. The 5 alternative answers were reordered for 3 of the 4 problems from each of the 21 problem types so that the correct answer would be in a different position for each of the 4 problems. To indicate the order in which subjects were to perform the transformations, two one-word transformation labels were printed at the top of the problem. For example, if *reflect, add* were printed above the analogy, subjects were supposed to perform the *reflect* transformation first and the *add* transformation second. All of the stimuli were presented on slides.

Design

Two sets of the 84 problems were constructed. They differed only in the order in which subjects were told to perform the transformations. The Set I stimuli are described below; the transformation labels were reversed for each problem in order to get the Set II stimuli. First, two problems from each pair of transformations were assigned to one transformation order and two were assigned to the other order such that all four instantiations of the two transformations would occur in each order for each pair of transformations. For example, in Set I, subjects performed the transformations for the four *move/size* problems in the following orders: *move top–bottom, smaller; move right–left, bigger; bigger, move top–bottom;* and *smaller, move right–left.*

The 84 problems were randomly assigned to four blocks of 21 according to the following four constraints: (a) Each block should contain one problem from each pair of transformations, (b) each transformation should occur approximately equally often first and second in each block, (c) in each block the transformations should be performed in the preferred and unpreferred orders (on the basis of the Experiment 1 results) for approximately equal numbers of problems, and (d) the correct answer should appear approximately equally often in each of the five positions. Four of the random orders for presenting the problems in Experiment 1 were chosen to be used with the four blocks of analogies in this experiment. The four blocks of problems were presented to subjects in four different orders according to a Latin square design.

Procedure

Before beginning the experiment, subjects were familiarized with the two instantiations of each transformation so that they would be able to understand the transformation labels on the problems. The experimenter stressed the importance of performing the transformations in the order indicated for each problem. Subjects were told that the order in which the transformations were presented would vary: Sometimes it would be the same as the order in which they would have chosen to perform the transformations and sometimes it would not, but they were always to perform the transformations in the order specified. Subjects completed 14 practice problems to familiarize themselves with the task. Slide presentation and data collection were performed by an Apple II Plus microcomputer. The slides were projected onto the wall about 6 ft (1.8 m) in front of the subject. Subjects responded by pressing one of six buttons on a response panel that rested on a small table in front of them. Subjects were allowed to use either hand to make their responses as long as they used the same hand throughout the experiment.

The following procedure was followed for all problems: First, a fixation point appeared, indicating the start of a problem. It remained in view until the subject pressed the button on the far right, at which point it was replaced with an analogy after a delay of 1 s. This slide showed the A, B, and C terms of an analogy and contained a question mark in the D box (as in the top part of Figure 2). It remained in view for a maximum of 30 s. Subjects were to mentally solve the analogy and then press the button on the far right again when they figured out the answer. Accuracy rather than speed was stressed. The time from onset of the analogy to the button press was taken as a measure of solution time. Subjects failed to respond within the allotted time on an average of only 1 problem per subject (53% of the missing times were from *rotate/reflect* analogies). The analogy slide was replaced by the alternatives slide 750 ms after the button press (or after the 30-s time limit if the subject failed to respond). When subjects found the answer they had mentally constructed, they responded by pressing one of the five buttons on the left, which were numbered *1* to *5*. The response to this slide caused a rating question to be presented after a 750-ms delay. Subjects were asked to rate on a 5-point scale how easy or difficult it was to perform the transformations in the order indicated. The scale went from 1 (*fairly easy*) to 5 (*fairly difficult*); these data were not informative because of a floor effect and thus will not be discussed. Subjects responded by pressing one of the numbered buttons. The fixation point then reappeared, indicating the start of the next problem.

There was a short break between each of the four blocks of slides, during which the experimenter changed slide trays. Subjects were given a longer break of several minutes between the second and third blocks, that is, about halfway through the experiment. Subjects were tested individually in 1-hr sessions.

Results and Discussion

Correlations With Experiment 1

In all analyses, solution times for problems on which subjects failed to indicate the correct answer were excluded. No subject was missing data for all four analogies from a given problem type. However, for a few subjects and problem types, this procedure did result in missing data for one of the two transformation orders. To facilitate analyses, missing data for one transformation order were replaced with the data from the other transformation order for that problem type. For example, if a subject's *rotate, move* time was missing, it was replaced with the *move, rotate* time. This procedure works against finding the predicted difference between transformation orders.

In order to compare results across Experiments 1 and 2, it is important to verify that the problems behaved similarly in the two experiments. That is, the order of the 21 problem types in terms of difficulty should be similar in the two experiments. We computed the Spearman rank-order correlation between solution times and number correct in this experiment with problem difficulty ratings and number correct in Experiment 1 (these data are shown in Table 1). The time and accuracy data from the present experiment were collapsed across the preferred and unpreferred transformation orders for each problem type. The analyses presented in the next section justify this analysis decision; they indicate that the effect of transformation ordering on both time and accuracy is constant across problem types. Both measures from the current experiment are highly correlated ($r_s = -.68, p < .001$).

The cross-experiment correlations clearly show that there is a stable ordering of transformation pairs. Problems judged to be more difficult in Experiment 1 were correctly solved less often in Experiment 2 ($r_s = -.67, p < .001$) and required more time to solve ($r_s = .94, p < .001$). Similarly, accuracy scores in the two experiments are very highly correlated ($r_s = .79, p < .001$). The only nonsignificant correlation is for accuracy in Experiment 1 with solution times from the present experiment ($r_s = -.27, p > .24$).

Table 1
Number Correct, Time, and Ratings for Each Transformation Pair on the Basis of the Data From Experiments 1 and 2 (Ordered by Decreasing Solution Time)

Transformation pair	Experiment 1		Experiment 2	
	Number correct (out of 48)	Difficulty rating (7-pt. scale)	Solution time (ms)	Number correct (out of 128)
Rotate/reflect	36	4.0	17,327	96
Move/rotate	41	3.2	12,616	112
Rotate/shading	44	3.0	12,530	116
Rotate/add	40	3.5	12,143	110
Move/reflect	41	2.9	11,447	114
Reflect/remove	47	2.6	11,226	123
Reflect/add	45	2.9	11,123	120
Rotate/size	40	2.8	10,774	115
Reflect/shading	44	2.8	10,453	116
Reflect/size	41	2.6	10,241	116
Remove/add	48	2.7	10,191	125
Rotate/remove	42	2.5	10,187	117
Move/remove	45	2.5	9,044	122
Add/shading	41	2.7	8,894	118
Move/add	48	2.6	8,479	126
Size/add	41	2.3	7,623	117
Move/size	43	2.3	7,344	119
Remove/shading	45	2.5	7,269	121
Move/shading	44	2.0	7,134	122
Remove/size	47	2.4	6,922	123
Size/shading	39	2.0	6,188	121

Consequences of Transformation Order

Solution times. Analyses of the consequences of transformation ordering were performed using only those pairs of transformations that were consistently ordered in Experiment 1. Each subject solved analogies using both transformation orders for each of these problem types. However, subjects solved any given analogy using only one order. That is, any particular analogy was solved in one order by subjects who received Stimulus Set I and in the opposite order by subjects who received Stimulus Set II. Because the effect of transformation order is expected to be small relative to other factors known to influence problem solution (e.g., encoding and inference), and because these other factors are very likely to make different problems involving the same transformations differ in difficulty, it is important to compare identical problems solved by subjects using different transformation orders. For example, it might take longer to perform Transformations X and Y (regardless of order) on a complex figure than on a simple figure. Comparing solution times for different transformation orders when the figures differ in this way would not constitute an adequate test of the effect of transformation ordering. Thus, although subjects performed all pairs of transformations in both orders, transformation order must be considered a between-subjects variable in the analyses. For a given analogy, different transformation orders are represented by different stimulus sets.

One further complication is needed. Subjects solved two of the four analogies for a given problem type using one transformation order and two using the other transformation order. For example, subjects who received Stimulus Set I solved *reflect/add* problems 1 and 2 using the preferred order and *reflect/add* problems 3 and 4 using the unpreferred order. In contrast, subjects who received Stimulus Set II solved problems 1 and 2 using the unpreferred order and problems 3 and 4 using the preferred order. It is clear that if we refer to problems 1 and 2 as Problem Set α and problems 3 and 4 as Problem Set β, then the predicted effect of transformation order appears as a Problem Set × Stimulus Set interaction: For Problem Set α, Stimulus Set I times should be shorter than Stimulus Set II times, whereas for Problem Set β, Stimulus Set II times should be shorter.

The data were analyzed by a 2 (stimulus set) × 2 (problem set) × 18 (problem type) analysis of variance (ANOVA). Significance levels for effects that include the subjects factor in the error term (i.e., certain main effects and interactions involving the within-subjects factors: problem set and problem type) are based on the Geisser-Greenhouse correction. There is a main effect of problem type, $F(17, 510) = 32.88$, $p < .001$. Solution times range from about 6.0 s (*size/shading* problems) to 12.5 s (*move/rotate* problems), with a mean of 9.5 s. The main effect of problem set is also significant, $F(1, 30) = 9.63$, $p < .005$, as is the Problem Set × Problem Type interaction, $F(17, 510) = 6.85$, $p < .001$. These results confirm the a priori prediction that the different problems in the two problem sets would not be equivalent in difficulty. The three-way interaction of Problem Type × Problem Set × Stimulus Set fails to reach significance, $F(17, 510) = 1.50$, $p > .15$, as does the main effect of stimulus set, $F(1, 30) < 1$, and the Stimulus Set × Problem Type interaction, $F(17, 510) = 1.15$, $p > .33$. Thus, we are free to examine the interaction of interest.

The results clearly confirm the hypothesis that more time is needed to solve analogies when the unpreferred transformation order is used, $F(1, 30) = 7.06$, $p < .02$, for the Problem Set × Stimulus Set interaction. The analogies were solved 413 ms faster when subjects performed the transformations in the preferred order as opposed to the unpreferred order (Ms = 9,267 and 9,680 ms, respectively). We are assuming here that subjects performed the transformations in the orders indicated. If subjects did not do so, it is reasonable to assume that switching to the opposite order would be more likely when subjects were told to perform the transformations in the unpreferred order. Because the decision to switch orders will take time, we would still predict longer solution times for the unpreferred orders. A third possibility is that solution times are longer for the unpreferred orders because we disrupted subjects' natural strategy rather than because those orders are inherently more difficult. Both of these alternative accounts of the data are consistent with the accuracy of subjects' reported transformation orders in Experiment 1. Note, however, that support for the veridicality of self-reports in this situation does not constitute general support for the veridicality of self-report data in other situations (see, e.g., Ericsson & Simon, 1984; Sternberg & Ketron, 1982).

Accuracy. For each subject, we had data on the number of analogies correctly solved for each problem set for each problem type. There were two analogies in each problem set for all problem types except *size/add-part, add-half/shading,* and *shading/add-part,* for which there was only one analogy per problem set. Subjects' scores for each problem set for these three problem types were multiplied by two in order to have all data on the same scale and to facilitate comparison of accuracy rates across problem types. Before performing the ANOVA, the within-cell variances were stabilized by taking the square root of each score plus one half, as suggested by Winer (1971). For the main hypothesis of a transformation order effect, we should again observe a Problem Set × Stimulus Set interaction.

Performance was very good overall (M = 93% correct), as might be expected given that accuracy rather than speed was stressed. However, the problem types did vary in difficulty, $F(17, 510) = 2.04$, $p < .05$. The (untransformed) scores range from 3.47 of 4 problems correct per subject (collapsed across transformation orders; *rotate/add* problems) to 3.97 problems correct per subject (*move/add* problems), with a mean of 3.71. Across all 32 subjects, these numbers correspond to 110 (86%) of the 128 *rotate/add* problems solved correctly compared with 126 (98%) of the *move/add* problems. The problem type effect does not vary with stimulus set, $F(17, 510) = 1.14$, $p > .33$. Consistent with the solution time analysis, the two sets of problems differ in difficulty across problem types, $F(17, 510) = 1.98$, $p < .06$. These variables do not further interact with stimulus set, $F(17, 510) < 1$, nor are the main effects of stimulus set or problem set significant, both $Fs(1, 30) < 1$. Finally, the important Problem Set × Stimulus Set interaction is again significant, $F(1, 30) = 6.60$, $p < .02$. As predicted, performing the transformations in the preferred order increases the likelihood of correctly solving an analogy (note, however, that the same caveats raised for the solution time results apply here also). Comparing error rates, 59% of the errors came from analogies for which subjects were told to use the unpreferred order, as opposed to 41% from problems solved using the preferred order.[1]

The solution time and accuracy data are consistent in demonstrating significant consequences of transformation order. The difference in accuracy is particularly meaningful because accuracy was stressed in the instructions at the expense of speed; thus, even when subjects are motivated to solve all problems correctly, the order in which the transformations are performed influences accuracy. Although both orders for performing two transformations are equivalent logically, they are not equivalent psychologically.

The Difficulty Hypothesis Reconsidered

As in Experiment 1, we computed the average number of errors for problems involving each of the eight transformations (again considering the two *add* transformations separately). The resulting difficulty order correlates .84, $p < .02$, with the Experiment 1 difficulty order. Because there were only minor differences between the two orders and because there is no reason to prefer one order over the other, we combined the two orders (by averaging the error rates for each transformation) in order to provide a better test of the difficulty hypothesis. The following order results: *rotate* (14.6% errors) > *reflect* (10.6%) > *size, add half* (10.3%) > *shading* (8.9%) > *move* (6.9%) > *add part* (4.9%) > *remove* (4.3%). This difficulty order is not significantly correlated with the performance order shown in Figure 3b ($r_s = .32$, $p > .10$; the critical value for $p < .05$ is $r_s = .74$). This finding constitutes another failure to support the transformation difficulty version of the working-memory hypothesis for problems involving two transformations.

Experiment 3

This experiment was designed to accomplish two objectives. First, we sought to replicate and extend the results of Experiment 1 by examining transformation application in analogies that required three transformations to be performed on a single figure. Although the performance order shown in Figure 3b is consistent with most individual orders, it is probabilistic rather than absolute; so a reversal of two adjacent transformations would not be unexpected. Second, we wanted to see whether ability differences and transformation difficulty would have a

[1] One might wonder whether the influence of transformation ordering on accuracy extends to an influence on the types of incorrect alternatives chosen. We analyzed these data for Experiments 1 and 3, both of which were conducted prior to the present experiment. In Experiment 1, there was some agreement in distractor choice, given an error, for a few problems. Across problems, however, there was no systematicity in the types of distractors chosen. In particular, there were no regularities as a function of transformation ordering. Experiment 3 was similar to Experiment 1 except that subjects solved three-transformation problems rather than two-transformation problems. In general, the errors in that experiment tended to result from the misapplication of the two orientation transformations. However, there were again no regularities as a function of transformation ordering. In light of these two sets of negative results, no in-depth analysis of the Experiment 2 error data was conducted.

greater effect on the order of transformation application with more difficult problems and a more heterogeneous sample of subjects.

Method

Subjects

The subjects were 59 University of Iowa undergraduates who participated in partial fulfillment of course requirements. Approximately half of the subjects were male, and half were female.

Stimuli

A set of 12 three-transformation problems was constructed so that each of the seven basic transformations was used four to six times, and the two instantiations of each transformation were used approximately equally often. Given that we could use the Experiment 1 results to predict partially the application order for this experiment, we tried to construct problems such that a given transformation would be performed in a variety of positions. For example, we predicted that *remove* would be performed first in one problem, second in two problems, and third in two problems. All pairs of transformations (except *rotate/reflect*) were used at least once, and most were used twice (including all pairs of transformations that are adjacent in the Figure 3a order). The following 12 transformation triples were used: *move/rotate/remove, move/reflect/remove, move/rotate/size, move/reflect/shading, move/shading/add-part, rotate/remove/add-part, rotate/size/add-half, rotate/shading/add-part, reflect/remove/size, reflect/add-half/shading, remove/size/shading,* and *size/add-half/shading.* The four distractors for each problem were constructed as follows: Three distractors were the result of applying two of the three appropriate transformations. The fourth distractor resulted from applying two of the transformations correctly and one of them incorrectly. Three different random orders for the problems were constructed according to the following two constraints: (a) Two consecutive problems could not have more than one transformation in common, and (b) no transformation could appear in more than two consecutive problems.

Design

The *transformations unknown* condition ($N = 29$) was a replication of the Experiment 1 solution condition using three-transformation problems. Because previous work (Mulholland et al., 1980; pilot testing) has shown that subjects have difficulty solving such problems, and because examination of transformation ordering depends on subjects marking the appropriate transformations, we included a second condition in which we could be fairly confident that subjects would use the correct transformations. In the *transformations known* condition ($N = 30$), we told subjects what the appropriate transformations were for each problem. To make this condition as similar as possible to the transformations unknown condition, we simply removed the lines next to all transformations in the list on the first page of each problem that were not needed for solution. We did not expect the application order to differ for the two conditions. However, we did expect to get more usable data from the new condition.

Procedure

All subjects completed the timed analogy test prior to solving the experimental problems. Subjects were given the same instructions for the experimental task as the solution subjects in Experiment 1 except for the following modifications: The transformations unknown subjects were told that all of the problems required three transformations. The transformations known subjects were told that in order to help them solve the analogies, we had indicated the three appropriate transformations for each problem by placing a line next to those transformations on the list on the first page of each problem. They were told to use the first half of the analogy (the A and B terms) to figure out how the three transformations were done, and then to number the transformations in the order in which they performed them on the C figure. Before solving the experimental problems, subjects solved three practice problems. All subjects were given feedback as to the correct answer for each problem, and those in the transformations unknown condition were also told the appropriate transformations. In giving this feedback, the experimenter was careful to stress that whatever order subjects performed the transformations in was okay. Subjects were then given as much time as they needed to solve the 12 experimental problems.

Subjects in the two conditions were tested separately in small groups ranging from 3 to 6 people per group. Each subject participated in a single session that lasted 45–60 min.

Results and Discussion

Examination of the number of problems on which subjects marked the correct transformations shows that the transformations known condition improved subjects' performance in this regard. All but 5 of the 30 subjects numbered the three transformations indicated for each of the 12 problems ($M = 11.6$). In contrast, the transformations unknown group averaged 9.8. No statistical comparison of these means was made because of the virtual lack of variability among subjects in the former condition.

One subject in the transformations unknown condition received a score of 2 on the timed analogy test, which is 9 points lower than the next lowest score from that condition. Because this subject did very poorly on all measures, the means reported below exclude the data from this subject. Subjects in the transformations unknown and transformations known conditions were comparable in analogy ability: Ms = 18.5 and 17.5, respectively, on the timed analogy test, $t(56) = 1.15$, $p > .25$; Ms = 9.5 and 9.9, respectively, on the experimental analogies, $t(56) = -0.98$, $p > .33$.

Although more usable ordering data was obtained from the transformations known condition, this did not have important consequences for the ordering of the transformations. As predicted, for none of the 12 problems did the distribution of transformation orders used in the two conditions differ reliably (on the basis of a chi-square test for each problem). Thus, the scaling analyses are based on data from the two conditions combined. These analyses included data from all problems for which the appropriate transformations were numbered, regardless of whether subjects later marked the correct answer to the analogy. For no problem did inclusion of the ordering data from subjects who marked an incorrect alternative change the most frequently used order from what it otherwise would have been.

A paired-comparison matrix was constructed for each subject on the basis of that subject's ordering of the 12 transformation triples. Because the two *add* transformations behaved differently in the first experiment, they were treated as separate transformations in the present experiment, yielding 28 pairs of transformations. As stated earlier, however, the *rotate/reflect* cell of the matrix was empty. In addition, four other cells were empty because most transformations were combined with only one of the two *add* transformations. The *add-half/add-part* combination also was not used.

The overall transformation order based on all 59 subjects is the best representation of the transformation scores for 51 of the subjects, and the average Spearman rank-order correlation between all pairs of subjects is .19, $\omega = .21$, $\chi^2(7, N = 59) = 84.83$, $p < .001$. The rank orders of the transformations in this solution are basically the same as in the solution for high-ability subjects described below. The only major difference between the three-transformation application order and the two-transformation application order from Experiment 1 (see Figure 3b) is that *move* comes after *rotate* and *reflect* instead of before. One minor difference is that *add half* and *size* are clearly separated in this experiment, but were not in Experiment 1. The Spearman rank-order correlation between the orders in which the transformations were performed in the two experiments is .91 ($p < .01$).

Although the observed three-transformation order replicates the two-transformation order, it is clearly less stable. One possible explanation for this is that high- and low-ability subjects performed differently. As in Experiment 1, ability was defined in terms of performance on the timed analogy test. Analysis of the test data again shows that subjects were misled by one of the distractors on the last problem. The revised 23-item test has a Cronbach's alpha reliability of .84 for these subjects. The correlations between overall performance and individual item scores range from .12 to .76 ($M = .48$). The item difficulties range from .12 to .98 ($M = .77$, $Mdn = .83$). Subjects in this experiment scored somewhat lower on the analogy test than did subjects in Experiment 1 (Ms = 17.7 and 18.7, respectively), $t(153) = 1.87$, $p < .07$. This was mainly due to greater differentiation among subjects at the low end of the distribution in this experiment.

Scaling analyses were performed separately on the data for high- ($N = 23$; test scores of 20–22), middle- ($N = 20$; test scores of 18–19), and low- ($N = 16$; test scores of 2–17) ability subjects. Note that 38% of the low-ability subjects in this experiment have lower scores than the lowest score observed in the solution condition of Experiment 1. The ability analyses show that the instability of the overall application order is in fact due to differing performance as a function of ability. The performance order for high-ability subjects is as follows: *rotate, reflect* > *move* > *remove* > *add half* > *size* > *shading* > *add part* (rank-order correlation with Figure 3b is .92, $p < .01$). This order is the best representation of the transformation scores of 22 of the 23 subjects, and, as indicated earlier, is basically the same as the overall order. The average intersubject rank-order correlation is .38, $\omega = .41$, $\chi^2(7, N = 23) = 65.77$, $p < .001$.

The story is quite different for the middle- and low-ability subjects. The middle-ability order is similar to the high-ability order ($r_s = .90$, $p < .01$), but the average intersubject correlation is only .15, $\omega = .19$, $\chi^2(7, N = 20) = 26.99$, $p < .001$. This order provides the best account of the transformation scores of 17 of the 20 subjects. The low-ability subjects do not really have a consistent ordering of the transformations. The best-fitting order accounts for only 11 of the 16 subjects and the average intersubject correlation is only .08, $\omega = .14$, $\chi^2(7, N = 16) = 16.02$, $p < .05$. Furthermore, to the extent that the transformations are ordered at all, the order is negatively (but not significantly) correlated with the other performance orders.

Thus, with more difficult problems and a wider range of ability, we find individual differences in ability affecting both the order in which mental transformations are performed during geometric analogy solution and the consistency with which the transformations are ordered. Although middle-ability subjects tend to perform the transformations in an order similar to that of high-ability subjects, they are much less consistent in their use of this order. Low-ability subjects do not appear to have a consistent order for performing the transformations.

The question remains, of course, as to what determines the order in which transformations are performed. Although transformation difficulty has been ruled out as a determinant of the two-transformation performance order, it may modulate performance on the more difficult three-transformation problems. In particular, the *move* transformation, a relatively easy one, is first in the order for two-transformation problems, but comes after the difficult orientation transformations for the three-transformation problems. Although the rank-order correlation between the difficulty order presented at the end of Experiment 2 and the three-transformation performance order is .52, this is not significant ($p > .10$; critical value for $p < .10$ is $r_s = .64$ and for $p < .05$ is $r_s = .74$).[2]

Given that the only difference between the three-transformation application order (excluding the low-ability subjects) and the two-transformation order is in the placement of the *move* transformation and that transformation difficulty is not significantly correlated with either performance order, there must be cognitive constraints other than difficulty operating to determine transformation ordering. The task-specific hypothesis that transformations are performed in the order in which they are initially identified also fails to account for the application order. Ability differences seem mainly to influence the consistency with which an order is followed, rather than the actual order itself, but only for the more difficult three-transformation problems. The next section proposes and explores a new explanation for the consistent performance order, namely that constructing a solution to these visual analogies is similar to constructing simple drawings.

Experiment 4

Performing mental transformations to construct the solution to a geometric analogy is often an imagery task. In the psychometric literature (see, e.g., Lohman, 1979; Snow, Kyllonen, &

[2] A difficulty ordering based on the error rates for the three-transformation problems was not used for several reasons. First, there are very few observations per transformation. In Experiments 1 and 2, there were six or seven observations per transformation, and each observation was based on data from either two or four analogies (usually four). In the present experiment there are three to six observations per transformation, and each observation is based on data from only a single analogy. A more serious problem is that in Experiment 3 the transformations were not combined factorially, as they were in the previous two experiments. This makes comparison of difficulty scores across transformations problematic.

Marshalek, 1984; Sternberg, 1977), geometric analogy tests have been found to load moderately to highly on the spatial visualization factor defined by tests such as paper folding, surface development, Minnesota paper form board, and block design. Geometric analogy tests also load moderately on a visual memory factor, as does paper folding (Lohman, 1979). This relation between geometric analogy solution and imaginal processing was highlighted in our studies. In particular, the way in which our analogy task was structured forced subjects to use the imaginal constructive matching solution strategy as opposed to the response elimination strategy (see Bethell-Fox et al., 1984).

In what order are operations performed during image construction? According to the theory of Kosslyn and his colleagues (Kosslyn, 1980; Kosslyn et al., 1984), overall shape should be ascertained before parts, but other common geometric analogy transformations are not ordered. For Piaget and Inhelder (1956, 1971), construction of mental images is closely related to action, in particular, drawing, a skill most people practice early in life. We propose that the ordering of mental transformations in analogy solution parallels the order in which the corresponding information is needed when planning and executing a simple drawing. Remember, however, that the transformations were derived from those found on psychometric tests and as such do not always have natural analogs in the drawing of simple figures. We will argue that in drawing, some kinds of information are needed prior to others, and will then present evidence for the proposed drawing order.

Consider the following thought experiment: How would you plan to draw a simple picture, such as a geometric figure? You probably begin with only a general idea of the outlines and proportions of the figure to be drawn; the exact location, orientation, color, and size are not yet specified. Before putting pencil to paper, you need to decide exactly where on the page the figure is to be located so that you know where to begin drawing. Next, you need to determine the exact orientation of the figure in order to know in what direction to draw the first line. At this point, having determined where to put your pencil and in which direction to draw, you can begin drawing; but you will not be able to proceed very far until you decide how long to make the line. Thus, the third decision you must make concerns the exact size of the figure. Now you can draw the entire outline of the figure. Internal and external details such as shading and small parts would be planned and drawn last. For example, think about how you would draw a zebra. After deciding its position and orientation, you would first draw an outline, and then add details such as the eyes, tail, and stripes. Whether you draw the shading before or after the small parts probably depends on the particular object being drawn, and, particularly, on whether the added parts are to be colored in or not.

Clearly, there are situations in which the above description is not valid. For example, if you have to draw a complex diagram in a very small space, size becomes more important and you will need to have more than just a vague idea about the figure's size from the very beginning. In this situation, there are additional constraints imposed on the drawing process because of the specific nature of the task. In contrast, our hypothesis centers on the drawing of simple figures or pictures in a relatively neutral context.

The hypothesized order of drawing decisions described above, *location* > *orientation* > *size* > *shading, small added parts,* is remarkably similar to the order in which the corresponding transformations are performed: *move* > *rotate, reflect* > *size* > *shading* > *add part* (see also Figure 3b). Note that the drawing order hypothesis does not make any prediction regarding the ordering of *rotate* and *reflect,* which in fact were unordered in Experiments 1 and 3. However, as suggested earlier, the parallel between construction of drawings and of analogy solutions is not complete. Although the *remove* and *add half* transformations are common in analogies, they cannot naturally be translated to a drawing strategy. Parts of figures are often erased from a drawing, but in the present task the part would not have been drawn in the first place. A hypothesis concerned with how people plan and execute drawings must be silent with respect to situations in which nothing is drawn. The *add half* transformation is also unusual in that when we think about adding parts to a figure being drawn, those parts are typically small relative to the main outline of the figure. With *add half,* the added part has to be just as large as the original figure. Thus it is not clear what the analog to the *add half* transformation would be in a drawing situation.

Some verification for the priority of certain information comes from the next experiment. Subjects were asked to plan a simple drawing and were queried about the order in which they would like to receive information about the object's location, orientation, size, parts, and shading.

Method

Subjects

The subjects were 48 Stanford undergraduates and psychology department staff. Of the undergraduates, 21 participated in partial fulfillment of course requirements; the remaining undergraduates and all of the staff members (i.e., 27 subjects) volunteered their time.

Design and Procedure

Subjects were asked to imagine that they were going to have to draw a simple figure (e.g., a picture of a small cane). They were told that in planning and executing such a drawing, there are (at least) five pieces of information they need to know: (a) the exact location of the figure on the piece of paper; (b) the exact orientation of the figure; (c) the exact size of the figure; (d) small, incidental parts to be added to the figure; and (e) how, if at all, the figure is to be shaded. These were presented and described in one of five orders (see below). Subjects were to imagine that we would give them each piece of information as they requested it. Then they were asked to number the five pieces of information in the order in which they thought those pieces of information would be needed in order to plan and execute a drawing.

The five pieces of information, and a brief description of each, were printed at the end of the instructions. There was a line next to each piece of information on which subjects were to write the appropriate number. Each piece of information appeared once in each position, resulting in five different forms of the questionnaire. In addition, each piece of information came before or after each other piece of information approximately equally often. Between 7 and 15 subjects responded to each of the 5 forms. The experiment took about 5 min.

Table 2
Number of Subjects (Out of 48) Who Indicated That Each Piece of Information in the Drawing Task Would Be Needed First Through Fifth

Type of information	When needed 1st	2nd	3rd	4th	5th
Location	**42**	3	3	0	0
Orientation	0	**32**	16	0	0
Size	6	13	**28**	0	1
Added parts	0	0	0	**33**	15
Shading	0	0	1	15	**32**

Results and Discussion

The ordering of the five pieces of information from first to last is as follows: *location* (87.5% of the subjects put this first) > *orientation* (67% put this second) > *size* (58%) > *small added parts* (69%) > *shading* (67%). The number of subjects placing each piece of information in each position is shown in Table 2. For each piece of information, more subjects indicated the position given in the order above than the next most frequently indicated position, as assessed by chi-square tests. By chance alone, only 20% of subjects should agree on any single position, yet even for size, which has the most variability, almost three times as many subjects as this agreed on its position. Thus subjects generally agree on the order in which they would prefer to receive different types of information during picture construction. An even more striking way to see this is to examine the number of subjects whose overall order conforms to the drawing hypothesis, that is, the number of subjects who put location first, orientation second, size third, and parts and shading fourth and fifth (or fifth and fourth). Because there are 120 different ways of ordering five items, and only 2 of these orders correspond to the drawing hypothesis, only 1.7% of subjects should confirm the hypothesis by chance. The data indicate that 58% of the subjects (i.e., everyone who put size third; 34 times as many people as would be expected by chance) support the drawing hypothesis in the ordering of all five pieces of information.

Thus, these data support the hypothesized order of decisions to be made during the planning and execution of simple drawings. Now we relate the observed order of drawing decisions to the order of transformation application. The drawing order results predict the following transformation order: *move* > *rotate, reflect* > *size* > *add part* > *shading.* Extracting the relevant transformations from Figure 3b, the observed order is *move* > *rotate, reflect* > *size* > *shading* > *add part.* The rank-order correlation between predicted and observed is .94, $p < .05$. The only discrepancy is in the ordering of *add part* and *shading.* Because we did not tell the drawing subjects whether the added parts could be shaded or not, they seem to have quite reasonably left open this possibility by putting parts before shading. If we had told them ahead of time that added parts would never be shaded (which is rather peculiar for a drawing task but was the case in our analogies), the order of parts and shading would likely have been more random, and might even have reversed. Thus, these data show a close correspondence between the order in which important pieces of information are needed during the planning and execution of simple drawings and the order in which transformations are performed in the solution of geometric analogies.

General Discussion

There are numerous ways of categorizing the types of problems or tasks routinely studied by psychologists or encountered in everyday life. Different categorization schemes are useful for different purposes. In this article, we considered a scheme whereby tasks are grouped according to the degree or type of option present in their solution paths. Some problems, such as Missionaries and Cannibals, have essentially only a single solution path. A single series of steps must be executed in a prescribed order if the solution is to be attained. Other problems, such as Rubik's Cube, have several completely different solution paths. Correct execution of any of these different procedures will lead to solution. Finally, there is a third category of problems for which there is a single set of solution components, but the components may be executed in any of several orders. We chose to study geometric analogies as a paradigm case from this last category. The main question of interest is whether solvers import cognitive constraints from other domains during solution of these problems, constraints that influence the otherwise unconstrained ordering of solution components.

Two experiments (1 and 3) demonstrated that the mental transformations used to solve geometric analogies (e.g., move, rotate, change size, and add a part) are performed in a consistent order despite the fact that the order is entirely optional (see Figure 3b). That is, even though the same figure results regardless of the order in which the transformations are performed, subjects select certain transformation orders over others at the application stage of solution. The preferred order, initially based on data from two-transformation problems, was replicated, except for the position of one transformation, with data from three-transformation analogies. The observed order does not simply represent subjects' unfounded preferences, for when subjects use the unpreferred order, solution time increases and accuracy decreases (Experiment 2). Thus, the preferred order is both psychologically compelling and cognitively advantageous.

Three classes of explanations for the observed transformation order were proposed initially: one based on individual differences; one derived from general constraints on information processing, in particular, working memory; and one based on task-specific constraints, in particular, perceptual ones. The individual-differences hypothesis received some support, but only from the solution of the more difficult three-transformation analogies. Middle-ability subjects tended to perform the transformations in an order similar to that of high-ability subjects, but they were much less consistent in their transformation ordering. Low-ability subjects did not seem to have a preferred ordering of transformation triples. These data suggest that one aspect of intelligence may be the ability to quickly discover and consistently apply a cognitively advantageous strategy (see, also, Campione et al., 1982; Sternberg, 1977). The individual-differences finding, however, gives no insight into the nature of such a strategy.

Because the transformations required to solve the analogies had to be performed with little benefit of external support, working memory is an important component of performance (Bethell-Fox et al., 1984; Hitch, 1978a, 1978b; Holzman et al., 1983; Kotovsky & Simon, 1973; Mulholland et al., 1980). The working-memory factor we considered was transformation difficulty. Of the variables shown to influence performance in inductive reasoning tasks, difficulty of the operations is the only one likely to affect performance in our task because the analogies were homogeneous with respect to the other variables. The task required subjects to apply either two (Experiments 1 and 2) or three (Experiment 3) transformations to a single figure in sequence in memory. Thus, an intermediate result had to be produced and retained on the way to constructing the final product of the transformations. This final figure then had to be stored temporarily in memory to match against the answer alternatives presented later. A reasonable strategy for reducing working-memory load might be to perform the transformations in decreasing order of difficulty, thus enabling the most difficult transformation to be carried out while there is a physical referent available to help guide that transformation. This hypothesis failed to receive support for either two- or three-transformation problems.

However, lack of support for this instantiation of the working-memory hypothesis does not necessarily imply that working-memory load was an unimportant factor in the solution of our analogies. Most of the variables that have been proposed in the past to determine working-memory load represent relatively general, task-independent information processing constraints. We turn now to task-specific factors to account for the order of transformation application. It is possible, however, that the effects of task-specific factors are mediated through the more general constraint of working memory. For instance, adopting a well-learned, readily-available procedure for solving a class of problems may reduce working-memory load relative to developing and/or executing a new procedure for each problem.

The first task-specific factor tested was a perceptual hypothesis with two parts: First, the order in which transformations are initially identified or inferred was assumed to be at least partially determined by perceptual factors such as transformation salience, although this assumption was not independently checked. Second, subjects were hypothesized to perform the transformations in the same order in which they identified them. This strategy reduces the number of problem solving steps by one, eliminating the step of independently determining the order in which to perform the transformations. The data failed to support this simple perceptual hypothesis. Although subjects tended to agree on a single order for performing transformations, very little agreement was observed in the order of identifying transformations.

Despite these results, the notion of a task-specific procedure that reduces working-memory load by constraining the order of operations is appealing. An alternative procedure is suggested by considering construction of the answer rather than identification of the transformations. There is evidence from other domains of a correspondence between mental and physical operations. In a study of expert and novice abacus users, Stigler (1984) found that experts performed mental arithmetic problems as if they were constructing the solutions on an abacus. Problems that required more operations on an abacus took experts longer to solve mentally. Constructing solutions to geometric analogies is an imagery task (Bethell-Fox et al., 1984), and psychometric studies have shown geometric analogy tests to be moderately to highly correlated with spatial visualization tests such as paper folding, Minnesota paper form board, cube comparison, and surface development (Lohman, 1979; Snow, Kyllonen, & Marshalek, 1984; Sternberg, 1977). Shepard and Podgorny (1978) have forcefully argued that many imaginal processes resemble perceptual processes: Mental rotation resembles the perception of a rotating object; mental size transformations resemble the perception of a physical object undergoing size changes; mental scanning resembles the scanning of a real surface (Kosslyn, Ball, & Reiser, 1978); and mental paper folding (using a surface development type of task) resembles physical paper folding (Shepard & Feng, 1972). These perceptually based cognitive processes underlie many of the individual mental transformations encountered in geometric analogies. Unfortunately for us, most theories of imagery (e.g., Kosslyn, 1980; Shepard, 1984) have been silent with regard to the order of these operations in image construction.

Forming a mental image is similar to constructing a picture, a task in which expertise is acquired at a tender age. Piaget and Inhelder (1956, 1971), in fact, have documented a close relation between drawing and mental imagery. Interestingly, Kosslyn, Holtzman, Farah, and Gazzaniga (1985) have recently investigated a split-brain patient (J.W.) whose right hemisphere had difficulty performing a task that depended on imaging letters. After practice first in drawing letters, and then in imagining drawing them, J.W. was able to perform the imagery task.

Logical considerations suggest that, in designing and constructing a picture, certain operations or decisions usually come earlier, and others later. Deciding where to start a line (location) would seem to precede a decision about the direction in which the line will be drawn (orientation), which in turn precedes a decision about the length of the line (size). After the general outline of a figure has been drawn, small parts and shading may be added. In the last experiment, subjects imagining drawing simple pictures tended to request this information in the order indicated above, providing some empirical support for the proposed drawing order. Thus, the order of performing transformations to solve geometric analogies parallels the order in which the corresponding information is needed in planning and executing a drawing. Returning for a moment to the idea that a task-specific procedure may reduce working-memory load by eliminating a step in the solution process, re-analysis of the Experiment 2 data showed that performing the transformations in drawing order reduced solution time by 578 ms.

Not only do construction of analogy solutions and drawings occur in similar sequences, but perceptual processing during object identification also seems to proceed similarly. The work of Treisman (Treisman, 1985; Treisman & Gelade, 1980) suggests that for complete identification of an object, all the object's features must be perceived to be in the same place. Similarly, Kubovy's (1981) theory of indispensable attributes posits the primacy of spatial location over all other dimensions except time. Thus, the location decision and the *move* transformation occur first. Rock (1973) has presented evidence that assigning an orientation to a figure is essential to identifying it, hence the orientation decision and the *rotate* and *reflect* transformations occur next. Studies by Rock, Halper, and Clayton (1972) have shown that figure contours are better encoded than internal details, suggesting that decisions and transformations affecting contours precede decisions and transformations affecting internal details. Thus, *shading* should come near the end, as it does. What is early in the identification and representation of objects seems to parallel what is early in the physical construction of drawings and the mental construction of the images that represent analogy solutions.

Upon initial examination of the geometric analogy task, it seems self-evident, to borrow a phrase, that all transformation orders are created equal; after all, the same figure results regardless of the order in which the transformations are performed. The data, however, suggest otherwise. Not only do problem solvers prefer certain orders over others, but the preferred orders result in faster solution times and fewer errors.

We chose to study geometric analogies as a paradigm case of problems whose solutions require the performance of several operations in an unconstrained order. Our results indicate that, because of the cognitive constraints solvers bring with them to the task, this optionality may be more theoretical than practical. We suspect that this type of finding is likely to obtain for other tasks in this category. For geometric analogies, the cognitive constraints seem to a large extent to come from mechanisms shared by imagery, drawing, and object identification. That the same types of operations have primacy in so many different situations suggests that geometric analogies are tapping into a pervasive way of organizing and operating on the world.

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