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Chapter 8
Embodied Cognition and Learning Environment Design

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Much learning that takes place through formal learning environments is of a fragile, shallow variety where students forget what they have learned soon after the end of the learning events (and the testing at the end) and does not get applied when relevant situations arise that are removed from the learning setting in time, space and conceptual context. The learning never seems to become a part of the way the student thinks about and interacts with the everyday world. Recent basic cognitive research in embodied or perceptually-grounded cognition provides a new perspective on what it means for what it means for learning to become more a part of the way students understand and interact with the world; further it provides guidance for the design of learning environments that integrate the learning with experiences that make it more meaningful and useable (Dewey, 1938).

Embodied Cognition

There are a variety of perspective on embodied cognition (e.g., Varela, Thompson and Rosch, 1991; Damasio, 1994; Semin and Smith, 2008) with more linguistic approaches focusing on the grounding of semantics in bodily metaphors (e.g., Lakoff and Johnson, 1999; Johnson, 1987; Gibbs, 2005) and more cognitive psychological ones focusing on evidence for modal (sensory) representations and mental simulations (e.g., Barsalou, 1999; Glenberg, 1997; and Pecher and Zwaan, 2005). The embodied or perceptually-grounded cognition perspective we will focus on here says that a full understanding of something involves being able to create a mental perceptual simulation of it when retrieving the information or reasoning about it (Barsalou, 2008, 2010; Black, 2010). Both behavior and neuroimaging results have shown that many psychological phenomena that were thought to be purely symbolic show perceptual effects. For example, property verification (e.g., retrieving the fact that a horse has a mane) was thought to involve a search from a concept node (horse) to a property node (mane) in a symbolic propositional network and thus the time to answer and errors was determined by how many network links needed to be searched and how many other distracting links were present. However, embodied cognition research shows that perceptual variables like size (e.g., more important propertyies are retrieved faster) affect verification times and errors (Solomon and Barsalou, 2004). Also, neuroimaging results (e.g., fMRI) show that perceptual areas of the brain (involving shape, color, size, sound and touch) also become active during this task, not just the symbolic areas (e.g., Martin, 2007). Thus, if one is familiar with horses and manes then doing even this simple property verification involves a perceptual simulation.

Even text comprehension shows spatial (perceptual) effects. For example a switch in point of view in a narrative creates longer reading times and more memory errors because the reader has to switch the spatial perspective from which they are viewing the narrative scene in their imagination. For example:

John was working in the front yard then he went inside.

is read faster than with a one word change that switches the point of view:

John was working in the front yard then he came inside.

(Black, Turner and Bower, 1979). Thus, when reading even this brief sentence the reader is forming a rough spatial layout of the scene being described and imaging an actor moving around it – i.e., this is a simple perceptual simulation.

Glenberg, Gutierrez, Levin, Japuntich, and Kaschak (2004) shows how to teach reading comprehension using a grounded cognition approach. These studies found that having 2nd grade students act out stories about farms using toy farmers, workers, animals and objects increased their understanding and memory of the story they read. Further, if they also imagined these actions for another related story after acting it out with the toys, they seemed to acquire the skill of forming the imaginary world of the story (Black, 2007) when reading other stories, and this increased their understanding and memory of these stories. Thus, this grounded cognition approach increased the students reading comprehension. These studies also seem to indicate that there are three steps involved in a grounded cognition approach to learning something:

1. Have an embodied experience
2. Learn to imagine that embodied experience
3. Imagine the experience when learning from symbolic materials

An Embodied Learning Environment Example in Physics

An example of using an embodied cognition approach to designing learning environment and the learning advantages of doing so is provided by the graphic computer simulations with movement and animation that Han and Black (in press) used in perceptually enhancing the learning experience. In learning a mental model for a system, students need to learn and understand the component functional relations that describe how a system entity changes as a function of changes in another system entity. Chan and Black (2006) found that graphic computer simulations involving movement and animation were a good way to learn these functional relations between system entities. Han and Black (in press) have enhanced the movement part of these interactive graphic simulations by adding force feedback to the movement using simulations like that shown in Figure 1. Here the student moves the gears shown in the middle by moving the joy stick shown in the lower left, and the bar graphics show the input and output force levels for the two gears. Allowing the student to directly manipulate the gears enhances the students' learning, and enriching the movement experience by adding force feedback increases the students' performance even more. Thus the richer the perceptual experience, and therefore the mental perceptual simulation acquired, the better the student learning and understanding.

Insert Figure 1 about here.

The following three major sections provide more detailed examples of using embodied cognition to design learning environments that improve student learning and understanding. The first uses the gestural-touch interface provided by the iPad to provide the embodiment needed to improve Young students' number sense and addition performance. The second looks at students learning geometry embodied in an agent spatially navigating an obstacle course in a game.   The third looks at student learning by embodying their understand in simple video games and robot programming.

Gestural Interfaces and Learning Environments

Gestural interfaces are also known as natural user interfaces and include two types: touch interfaces and free-form interfaces. Touch use interfaces (TUIs) require the user to touch the device directly and could be based on a point of single touch (i.e., SMART Board) or multi-touch (i.e., SMARTtable/iPhone/iPad/Surface). Free-form gestural interfaces do not require the user to touch or handle the device directly (e.g., Kinect Microsoft project). The mechanics of touch screens and gestural controllers have at least three general parts: a sensor, a comparator, and an actuator. Saffer (2009) defines gesture for a gestural interface as any physical movement that a digital system can sense and respond to without the aid of a traditional pointing device, such as a mouse or stylus. A wave, a head nod, a touch, a toe tap, or even a raised eyebrow can be a gesture. These technologies suggest new opportunities to include touch and physical movement, which can benefit learning, in contrast to the less direct, somewhat passive mode of interaction suggested by a mouse and keyboard. Embodied interaction involving digital devices is based on the theory and body of research of grounded cognition and embodiment. The following sub-sections review evidence from studies on embodiment, physical manipulation, embodied interaction, and spontaneous gestures that support the theory of how gestural interface can promote thinking and learning. These are followed by a study conducted by Segal, Black, and Tversky (2010) about the topic.

Action Compatibility Effect

Bodily rooted knowledge involves processes of perception that fundamentally affect conceptual thinking (Barsalou, 2008). Barsalou and colleagues (2003), who have conducted extensive research in the field of grounded cognition and embodiment, found that there is a compatibility effect between one's physical state and one's mental state. This means that an interface that is designed to take an advantage of embodied metaphors results in more effective performance. For example, they found that participants who were asked to indicate liking something by pulling a lever towards them showed a faster response time than those who were asked to indicate liking by pushing the lever away. These findings have implications for the design of learning environments.

Physical Manipulation and Learning

Some educational approaches, such as the Montessori (1949/1972) educational philosophy, suggest that physical movement and touch enhance learning. When children learn with their hands, they build brain connections and knowledge through this movement. Schwartz and Martin (2006) found that when children use compatible actions to map their ideas in a learning task, they are better able to transfer learning to new domains. For example, children who had only a beginner's knowledge of division were given a bag containing candy and asked to share it with four friends. Children were asked to organize piles of candy into various groups (i.e., four equal groups). The other group of children solved the problem using a graphical representation (i.e., drawing pictures of the candy to be shared). Children who learned through complementary actions were in a better position to solve problems of division in arithmetic. Physical manipulation with real objects has also been proven effective with children as young as preschool- and kindergarten-age (Siegler & Ramani, in press). In this study, using linear number board games, children who played a simple numerical board game for four 15-minute sessions improved their numerical estimation proficiency and knowledge of numerical magnitude.

Embodied Interaction and Learning

Embodied interaction involves more of our senses and in particular includes touch and physical movement, which are believed to help in the retention of the knowledge that is being acquired. In a study about including the haptic channel in a learning process with kinematics displays, Chan and Black (2006) found that the immediate sensorimotor feedback received through the hands can be transferred to working memory for further processing. This allowed better learning for the students who were in the direct manipulation animation condition, essentially enabling the learners to actively engage and participate in the meaning-making journey. In a study that incorporates the haptic channel as force feedback to learn how gears operate, Han and Black (in press) found that using three sensory modalities, and incorporating tactile feedback, helped participants efficiently learn how simple machines work. Furthermore, the haptic simulation group outperformed the other group not only in the immediate posttest, but also in the near transfer test, meaning that effectiveness of this embodied experiences with haptic simulation was maintained during reading instructional text.

Do Spontaneous Gestures Reflect Thought?

According to theories of embodied cognition (Barsalou, 1999; Glenberg, 1997), concepts are primarily sensorimotor; thus, when speakers activate concepts in order to express meaning, they are presumably activating perceptual and motor information, just as comprehenders do when they activate meaning from language input. In theory, then, language producers must start with sensorimotor representations of meaning, just as language comprehenders end there. Hostetter and Alibali (2008) claim that these sensorimotor representations that underlie speaking are the basis for speech-accompanying gestures.

There is a growing body of research regarding spontaneous gestures and their effect on communication, working memory, information processing, learning, mental modeling, and reflection of thought. Goldin-Medow (2009) found that gesture plays a role in changing the child's knowledge; indirectly through its effects on the child's communicative environment, and directly through its effects on the child's cognitive state. Because gestures reflect thought and are an early marker of change, it may be possible to use them diagnostically, which may prove useful in learning and development. In a study on how gestures could promote math learning, it was found that requiring children to produce a particular set of gestures while learning the new concept of grouping strategy helped them better retain the knowledge they had gained during the math lesson, and helped them to solve more problems.

Schwartz and Black (1996) argued that spontaneous hand gestures are “physically instantiated mental models.” In a study about solving interlocking gear problems, they found that participants gestured the movement of the gears with their hands to help them imagine the correct direction of the gears, gradually learning to abstract the arrhythmic rule for that. In a study about mental representations and gestures, Alibali et al. (1999)

found that spontaneous gestures reveal important information about people's mental representations of math-based problems. They based their hypothesis on a former body of research that showed that gestures provide a window into knowledge that is not readily expressed in speech. For example, it may be difficult to describe an irregular shape in speech but easy to depict the shape with a gesture. The authors hypothesized that such mental models might naturally lead to the production of spontaneous gestures, which iconically represent perceptual properties of the models.

Gestural Interfaces and Spontaneous Gestures

If spontaneous gestures reflect thought, could it be that choosing well designed gestures (for gestural interface) could affect the spatial mental models of subjects? Hostetter's and Alibali's (2008) theory of Gestures as Simulated Action (GSA) suggests that gestures emerge from perceptual and motor simulations that underlie embodied language and mental imagery. They provide evidence that gestures stem from spatial representations and mental images, and propose the gestures-as-simulated-action framework to explain how gestures might arise from an embodied cognitive system. If gestures are simulated actions that result from spatial representation and mental imagery, it is very likely that asking users to perform one gesture versus another could affect users' mental operations to solve the problem in different ways.

Spontaneous gestures are being adopted by gestural interface designers in order to incorporate more natural and intuitive interactions. There are four types of spontaneous gestures: deictic, iconic (show relations), metaphoric (more abstract), and beat (discourse). Deictic gesture, such as pointing, is typically used for gestural interfaces. Iconic and metaphoric types of gesture are also very common to adopt for gestural interfaces, and usually indicate a more complex interaction. Using a familiar gesture (from everyday language) to interact with interfaces could ease the cognitive load of the user. It creates a more transparent interface and natural interaction with the computer.

Congruent Gestures Promote Performance

Segal, Black, and Tversky (2010) explored the compatibility of gestures designed for gestural interfaces with the digital representation of the mathematical concepts of counting, addition, and number line estimation. By simulating the mental operations needed to solve the problem with the correct gestures, learners constructed better spatial mental models of these mathematical procedures. When mapping gestures to the learned concept, one enhances the simulation for the mental operations, which needs to be constructed to solve a problem. The embodied metaphor, the gesture, represents the operation that needs to be mapped to the mental operations.

Can action support cognition? From a grounded cognition perspective, the use of gestural interfaces (such as multi-touch like the iPad) versus traditional interfaces (such as monitor-mouse) should yield better learning with computers. This question is addressed by observing children's performance in arithmetic and numerical estimation. Arithmetic is a discrete task, and should be supported by discrete rather than continuous actions. Estimation is a continuous task, and should be supported by continuous rather than discrete actions. Children either used a gestural interface or a traditional interface. The actions either mapped congruently to the cognition or not. If action supports cognition, performance should be better with a gestural interface when the actions map conceptually to the desired cognition, Gestural Conceptual Mapping.

Direct Manipulation: Gestural Conceptual Mapping

Marshall (2007) states that there is a gap in the existing research on tangible interfaces and learning. He claims that there is no research on how users abstract the underlying rules or laws of a domain, and how different levels of representation become integrated within the design. The gap, theoretically speaking, is about how the structure of the learning domain can be represented by the interface. The following case study explores the gap and defines it as Gestural Conceptual Mapping. The term Gestural Conceptual Mapping is used to convey the mapping between the representations of the physical embodied metaphor (the gesture), to the digital representation of the learned domain. This term is one of three properties of direct manipulation. It is a new term that Segal, Black, and Tversky (2010) define, explore, and focus on, in the design and use of gestures within interfaces to promote thinking and support better learning.

Segal, Black, and Tversky (2010) explored the compatibility of the learned concept 'visualization' (digital representation) with the physical representation of the gesture, and with the internal representation of the learned concept. Using a specific gesture to illustrate the learned concept helps the student construct a better mental model of the learned concept. For example, tapping with a finger on a virtual block or clicking with a mouse on a virtual block to count and add up are gestures that are congruent with the discrete representation of counting. In contrast, sliding the finger vertically over a series of blocks or dragging a mouse on a series of blocks to count them are continuous movements that are not congruent with the discrete procedure of counting. In other words, both the digital representation of the content and the gestures need to be compatible with the learned concept. Therefore, there must be compatibility between the external representation of the content and the internal representation the user constructs. This compatibility supports the user's mental imaging and allows for the construction of better mental models. In order to achieve this compatibility, designers should find the compatible embodied metaphor that would best illustrate the learned concept. The embodied metaphor is the type of gesture chosen by the designer to manipulate the educational content on the screen.

Direct Manipulation: Haptic Channel, Sensorimotor Input

Direct manipulation has been defined by Shneiderman (1983) as the ability to manipulate digital objects on a screen without the use of command-line commands (i.e., dragging a file to a trash can instead of typing "del"). Direct manipulation in the HCI field has been consistently changing over the past few years. This is a result of a boom in the development of new technologies and innovative interfaces, which have taken direct manipulation to another level. This is especially true for touchscreen and free-form gestural interfaces that do not require external control devices (i.e., mouse) to manipulate objects on the screen. Instead, they utilize the user's own body to manipulate objects on a screen, changing the level of direct manipulation.

Research has shown that physical manipulation could enhance the processing of abstract content and the comprehension of learned concepts. Based on this body of research, Segal, Black, and Tversky (2010) showed that gestural interfaces that incorporate the haptic aspect of touching the interface and manipulating the objects by using sensori-motor input could benefit users' comprehension of learned concepts. They hypothesized that by touching the objects on a screen directly with a finger/fingers, participants help themselves process abstract content and build internal representations that are more accurate. Touching the objects on a screen directly, with our body, rather than having a control device such as mouse or even stylus, could enhance the haptic channel experience and make the learning experience more direct and integrated with the content. It is a more concrete experience that could support young children's internal representations of learned concepts., as indicated in the study described next.

Congruent Gestural Interface Experiment

Participants. The researcher recruited 107 subjects (60 boys and 47 girls) from 1st and 2nd grade. Children were recruited from two after-school programs in public schools in a low SES area of New York City.

Materials. Two learning tasks with virtual manipulatives were given to subjects to examine the effect of high direct manipulation provided by gestural interfaces versus traditional interfaces. Two educational applications were developed to allow interaction and learning with two math concepts. The learned concepts explored were 1) discrete-change problems that focus on change over a series of steps, such as counting blocks, versus 2) continuous-change problems that focus on change over a single, non-partitioned event, such as number-line estimation. For the discrete-change problem, the tasks were counting and addition; for the continuous-change problem, the task was number estimation on a number line. The gestural interface was a 10-inch multi-touch iPad device by Apple, and the traditional interface was a Macintosh Macbook Pro laptop by Apple, which requires the use of a mouse. Software developed by the experimenter recorded the child's answers. In order to accurately record all children's strategies, the experimenter marked the strategies chosen by the child on a check box strategies list.

Variables and design. This was a 2 by 2 between subjects design. The children were randomly assigned to one of four conditions. These conditions were: 1) the haptic, gestural conceptual mapping condition, 2) the haptic non-gestural conceptual mapping condition, 3) the non-haptic gestural conceptual mapping condition, and 4) the non-haptic, non-gestural conceptual mapping condition. The direct manipulation was examined in both tasks and included two direct manipulation properties:

1. Gestural Conceptual Mapping: Mapping the gesture to the learned concept. It refers to the mapping between the information carried in the physical and digital aspects of the system. Using congruent gestures versus non-congruent gestures to support cognition.
2. Haptic Channel: Adding the haptic channel to perform these tasks, such as physical manipulation of the interface. This integrates the level of sensorimotor input (mouse versus touch).

Counting and Addition Task: Discrete Procedure. Children were required to solve 10 additions problems by working on a virtual manipulatives interface that showed virtual blocks arranged in side-by-side piles of two 10-block towers. The addition problems ranged from 1 to 20, such as, 6+7=? 2+9=? (see Figure 2). The computer narrated the questions so children did not need to recognize the symbols.

Haptic Channel Variable: Counting and Addition Task. The first variable compared use of the haptic channel (e.g., tapping with a finger on a multi-touch screen [iPad] to fill in digital blocks in a bar chart, performing addition) to use of the non-haptic channel (e.g., filling in the digital blocks by clicking them with a mouse via a traditional interface) (see Figure 2).

Gestural Conceptual Mapping Variable: Counting and Addition Task. The second variable compared the use of Gestural Conceptual Mapping to the use of Non-Gestural Conceptual Mapping, both on the multi-touch screen (iPad). This explored the representation of the gesture to support the mental model of discrete counting. In one condition, children tapped with their finger on each digital block in a bar chart to highlight the block's color, then performed addition of both columns. This is a gesture that is conceptually mapped to the discrete concept of counting. In the other condition, children tapped on the numbers under each column of blocks (not on each block) and this automatically highlighted the colors of the blocks, which is not conceptually mapped to the discrete concept of counting (see Figure 2).

Number-line Estimation Task: Continuous Procedure. The second task of the number-line estimation was chosen to benefit the procedure of a continuous concept, such as magnitude of number-line. Number-line estimation requires translating a number into a spatial position on a number line, or translating a spatial position on a number line into a number. As noted in Siegler and Booth's (2005) review of the estimation literature, numerical estimation is a process of translating between alternative quantitative representations, at least one of which is inexact and at least one of which is numerical. Number line estimates correlate substantially with other measures of numerical magnitude knowledge, such as magnitude comparison and numerical categorization (Laski & Siegler, 2007).

In the present study, children were required to estimate 23 numbers (1-100) on a virtual number line (see Figure 3). The computer narrated the questions so children did not need to recognize the symbols. Prior to the task, the experimenter asked the child to show her if there was zero on the number-line, and if there was the number 100, to make sure the child recognized the numbers. The experimenter explained the task by saying, ‘‘a number line is a line with numbers across it. The numbers on the line go from the smallest number to the largest number, and the numbers go in order, so each number has its very own spot on the number line.’’ After each answer, the child received an animated feedback with the numbers appearing on th e number line from left to right, up to the correct value.

Haptic Channel variable: Number-line Estimation Task The first variable compared use of the haptic channel to use of the non-haptic channel in a continuous number-line task. In the haptic channel condition, using a multi-touch screen (iPad), the child slid his or her finger horizontally on the number line to estimate numbers; in the non-haptic channel condition, using a traditional (mouse) interface, the child dragged the mouse horizontally on the number line to estimate numbers.

Gestural Conceptual Mapping Variable: Number-line Estimation Task. The second variable compared use of gestural conceptual mapping to use of non-gestural conceptual mapping, both on the multi-touch screen (iPad). The number-line estimation task explored the compatibility of the gesture, to the mental model of a continuous concept. In one condition, the child tapped on the screen to estimate numbers on the number line (discrete gesture); in the other, the child slid his or her finger horizontally (continuous gesture) to reach the number (see Figure 2). The sliding gesture, in that case, is mapped conceptually to the concept of continuous magnitude of a number line. It simulates the mental operation of increasing something (i.e., a number line bar) continuously.

Insert Figures 2 and 3 about here.

Children in the Haptic, Gestural Conceptual Mapping condition group had the best performance across both tasks. Thus the children who moved their finger across the iPad screen to indicate numerical magnitudes and tapped on stacked blocks to indicate addition performed the best. They had the fewest errors on both numerical estimation and addition problems. Children who were in the Haptic Channel (iPad touch) condition used an advanced strategy significantly more times to solve the addition task. This means that the touch screen provided a better virtual environment for advanced strategies. This advanced strategy is the "count on" strategy. Children in the Haptic Channel (touch) condition outperformed the children in the non-Haptic Channel (mouse click) group in the use of this strategy. The children in the Haptic Channel condition, both in the TC (Touch, Conceptual condition) and TNC (Touch, non-Conceptual condition) used the advanced strategy "count on" significantly more times (between 3-5 times) than the children in the non-Haptic Channel (mouse) condition (less than 3 times).

These findings suggest evidence for the importance of designing gestures congruent with the learned concept. This means that actions affect performance and that congruent gestures are important for cognition and learning, especially when combined with the haptic channel (touch condition), but not only then. Congruent gestures are also effective in the non-haptic condition (mouse condition) facilitating better performance. The best performance was found when the touch screen and the congruent gestures were combined. The findings also suggest that the Haptic Channel allows better use of strategies (i.e., children constructed better mental models), providing evidence that touch-based interfaces could benefit thinking and learning.

Embodied Cognition, Gaming and Robotics

Learning Geometry with an Agent Spatial Navigation Game

The number line estimation example described in the preceding section demonstrates how embodied interaction affects conceptual representation for basic mathematical principles. The perceptual-motor basis of early cognitive development is, of course, a prominent feature in Piaget's theory (e.g. Piaget, 1954). Yet, the embodied perspective further asserts that "abstract" thought, associated with higher levels of development, also shares a perceptual-motor basis. In the case of geometry, where researchers have frequently posited series of stages or levels to account for increasing abstraction (e.g. Piaget & Inhelder, 1967; Piaget, Inhelder, & Szeminska, 1960; van Hiele, 1986), the embodied perspective entails that performance across tasks, both simple and complex, is based on physical interaction with the environment and the corresponding mental representations formed through those interactions. How then does embodied cognition explain behaviors that appear to reflect abstract, symbolic thought?

First, we must understand the source of geometric knowledge. Clearly those processes engaged in number line estimation are relevant in geometry. Furthermore, Spelke, Lee, and Izard (2010) claim that the innate human navigational and object perception abilities represent core systems, upon which Euclidean geometry may emerge. Object perception, is particularly relevant as young children may be introduced to geometric figures just as they might be introduced to any other object (e.g. physical objects and their corresponding names). This can be seen in young children who base their source of reasoning in identification and sorting tasks on a shape's "holistic" appearance (Clements, Swaminathan, Hannibal, & Sarama, 1999). The challenge with this approach is that children are often exposed to a limited number of exemplar figures, generating a sparse mapping between shape names and associated figures. For example, Clements et al. (1999) found an inverse u-pattern in triangle identification such that only very young children and older children correctly identified non-prototypical, scalene triangles as triangles. In this case, repeated exposure to prototypical, equilateral triangles produced a misconception about the meaning of the word "triangle".

A clear remedy for this type of misconception is to provide children with a wider range of curricular materials. Yet, geometric thinking requires more than just a large visual vocabulary of shapes. A child's success distinguishing a trapezoid from a parallelogram, for example, does not entail that he or she understands the defining features of the shapes or how they relate to each other. Rather, children – and adults – are likely to attend to perceptually salient, but formally irrelevant features of shapes.

For example, in a classic demonstration by Mach (1886/1959), adults may be prompted to perceive a shape as either a square or diamond, depending on its orientation. Similarly, in a study analyzing the perceptual similarity of four-sided figures participants' judgments appeared to be based on factors of "dispersion" (regularity), "elongation", and "jaggedness" (Behrman & Brown, 1968). Likewise, Shepard and Chipman (1970) found similar dimensions in participants' categorizations of U.S. state shapes. While these features are clearly relevant to the perception of commonplace objects (e.g. jagged objects can cut), they are only partially related to formal geometric properties of objects (e.g. acute angles). By applying terms like "slanty", "pointy", and "skinny", young children's verbal reasoning about shapes often reflects these informal characteristics (Clements et al., 1999).

How, then, may children's conception of shapes be guided towards more formal elements of geometry? Direct verbal instruction of shape definitions is a common, if unsatisfactory, method (Clements & Battista, 1992). A child may remember, for example, that a parallelogram has parallel sides. But, would he or she be able to recognize parallel lines in another figure, such as a square oriented as a "diamond"? Rather, the child must develop a spatial understanding of geometric properties (such as parallel lines) that is independent of any specific figure. From this perspective, a mature mental representation integrates, or blends, both a general sense of what a shape looks like and independent spatial representations for its properties (Lakoff & Nuñez, 2000). Developing this kind of complex representation requires both tools to ground individual mathematical concepts in spatially-meaningful representations and an environment to facilitate their integration. To implement this framework we developed a digital learning environment, in the form of a game, in which children construct polygons to serve as a path for an agent navigating an obstacle course. The obstacle course includes both "dangers", i.e., grid squares through which the path may not pass, and "goals", i.e., squares through which the path must pass. The layout of the obstacle promotes the construction of a specific geometric shape, such as a square. This may be achieved by either placing danger objects that either circumscribe or inscribe the intended path, or (more directly) by placing goal objects along the intended path.

The child proceeds by first viewing the obstacle course, attempting to imagine a potential path, and then constructing the path from memory on an empty grid. The child constructs the polygonal path by iteratively plotting sides and angles through direct manipulation of the mouse. Upon closing the figure, the child may then drag-and-drop vertex points to achieve greater precision (see figure 4).

By providing a variety of obstacle courses he child may be exposed to a wide range of geometric shapes that he or she has constructed. Yet, as described above, exposure is insufficient to promote higher-level thinking. Rather children must understand the properties which determine polygon class – e.g., congruency, parallelism, and right angle. To provide a spatial-grounding for these concepts we depicted each with a "hand metaphor", inspired by previous work demonstrating that children may spontaneously use their hands to model geometric properties, such as right angles (Clements & Burns, 2000).

During a "property validation" phase (see figure 5) the child is instructed to verify that a given number of sides or angles meet a specific property criteria, while a pair of virtual hands modeled the process. For parallelism two hands move in parallel at the same slope as a chosen side and are matched for slope against the second chosen side. For congruency two hands mark the distance of one chosen side and are matched against the length of the second chosen side. For right angles two perpendicular hands are matched against the internal angle at the chosen vertex. If the polygon does not meet property criteria the child returns to the adjustment phase. If the polygon successfully meets property criteria the child proceeds to testing on the obstacle course.

We tested this design with an afterschool class of twenty fourth grade students from a low income population. Ten children were randomly assigned to the experimental condition in which they performed a series of shape construction tasks with the software described above. For comparison, the other ten children were assigned to a control group, in which the property validation phase was removed, but all other aspects of the task remained the same. Therefore, these children were not exposed to the "hand metaphors" and received no feedback about the validity of their figures based on class-defining properties. In both conditions all children proceeded through a series of twenty-two construction tasks, in three units focusing on parallel lines (trapezoids and parallelograms), congruent adjacent sides (kites and rhombi), and right angles (rectangles and squares).

Following each unit the child was assessed with a shape identification task, targeting trapezoids, parallelograms, rhombi, isosceles trapezoids and triangles (mixed), rectangles, and right trapezoids and triangles (mixed). For each class, a single prototypical shape was constructed (e.g., a 2x1 rectangle oriented with its longer side parallel to the ground). This prototype was then altered on dimensions that changed its shape identity (e.g., a nearly rectangular parallelogram), and dimensions that did not change its shape identity (e.g., elongating or rotating the rectangle). In each trial the child was shown four shapes, and asked to identify the two valid shapes.

The results show that the median number of trials in which the child selected both correct shapes was greater for children in the experimental than in the control condition, for each shape type. On the other hand the children in the control group were more likely to choose only one correct shape. Therefore, children in the control group were drawn towards the shapes that were visually similar to prototypes, yet class invalid (e.g., the nearly rectangular parallelogram), while children in the experimental group were more likely to overlook these irrelevant perceptual features in favor of class-defining properties (e.g. the two shapes with right angles).

From an outside perspective the difference in the two conditions might imply greater abstract reasoning by those in the experimental condition, and more reliance on perception by those in the control condition. This difference could be interpreted as evidence for a general concrete-to-abstract shift in development, typical of stage-based theories. However, as details of the intervention reveal, better performance in the experimental condition was promoted through embodied interaction. Rather, than abandoning concrete representations the children reorganized their representations to integrate (or blend) more normatively meaningful, yet perceptually accessible components. We suggest that the development of higher-level mathematical skills, in general, reflects this reorganization of embodied representations. While some proportion of mathematical activity may simply be rote symbol manipulation, to understand how, when, and why to apply these procedures mathematical concepts must be grounded in a deeper understanding.

Insert Figures 4 and 5 about here.

Learning Through Embodying in Video Game and Robot Programming

Recent research we have done seems to indicate that learning of abstract computational concepts can be improved when embodied instruction is grounded in familiar actions and scenarios (Fadjo & Black, 2011). In this research, we were interested in whether middle school subjects who learned abstract computational constructs through physical and imagined grounded embodiment would implement more mathematical and computational constructs in the individual video game artifacts than those who learned the same constructs without the aid of physical embodiment. From a cognitive perspective, we were primarily interested in whether providing the formal instruction of abstract concepts through action and perception, both high-level cognitive constructs, would have a positive effect on the structures used to define the artifacts. To explore a grounded approach to the instruction of computational and mathematical thinking, we devised a curriculum where subjects received explicit instruction on Code Literacy (Fadjo, Black, Chang, & Lee, 2011; Fadjo, Black, Chang, & Hong, 2010) that would then provide a sufficient foundation upon which to explore Direct Embodiment (Fadjo et al., 2009, 2010) during Imaginary Worlds (Black, 2007) Construction.

For the most recent experiment we explored the effects of Instructional Embodiment on mathematical and computational thinking in video game design. Instructional Embodiment is the use of action and perception for the development of understanding and comprehension of concepts (abstract or concrete) through direct, surrogate, augmented, or imagined embodiment within a formal instructional setting. Unlike other pedagogical frameworks where an instructor may solely model, or embody, the concepts or principles she or he wishes to teach, Instructional Embodiment is the use of embodiment as an engaging activity for the student that may be modeled by the teacher, but is fundamentally designed to engage the student in a sequence or system of movement, imagination, and exploration. Seminal work by Seymour Papert and colleagues during the late 60s to mid 80s addressed a similar principle wherein the student used ‘feel’ and aesthetics with motion and augmented supports, such as an anthropomorphized robot, to learn (and ‘do’) geometry through Logo (Papert, 1976, 1980; Minksy, 1970). Indeed, Papert promulgated the theory that in order to understand geometry on must ‘do’ geometry, or mathematics for that matter, and in doing so is thinking as a mathematician would think. Similarly, we defined a framework of Instructional Embodiments that can be used in the classroom setting.

The Instructional Embodiment Framework (IEF – see Figure 6) is composed of two main categories, physical and imagined embodiment. A physical Instructional Embodiment may be Direct, Surrogate, or Augmented. Direct Embodiment (DE) is when the learner physically enacts a scenario using his or her body to enact statements or sequences. Surrogate Embodiment (SE) is physical embodiment that is controlled by the learner whereby the manipulation of an external ‘surrogate’ represents the individual. Augmented Embodiment (AE) is the use of a representational system, such as an avatar, in conjunction with an augmented feedback system (such as Microsoft’s Kinect and display system) to embed the embodied learner within an augmented representational system. Recent technological advances have made it possible to capture the entire human figure and embed him or her within a virtual space where the learner is not removed from reality (as was and is often characterized by virtual reality systems), but, rather, instantiated through a representational avatar who, in turn, becomes the augment of the learner during an instructional phase.

In addition to the physical Instructional Embodiments within IEF, an individual also embodies action and perception through imagination. Imagined Embodiment is characterized as the mental simulation of physically embodied action that is either Explicit (EI) or Implicit (II). Glenberg and colleagues work on physical and imagined simulation showed that, while physical manipulation increases free-recall facilitation, it was physical manipulation in concert with imagined manipulation that led to the significant learning-strategy maintenance of indexing novel terminology (2004). We believe that embodied cognition in a learning environment must first be physically enacted (either through Direct, Surrogate, or Augmented Embodiment) as a full perceptual experience, then the learning activity is maintained through imagined embodiment (typically as Explicit Imagined Embodiment (EI)) before finally resulting in tasks where transfer of learned content can occur. We also believe that cognition must be grounded, but not necessarily situated, within various contexts in order to be effective within an embodied learning environment design.

Grounding cognition involves both environment and body as "external informational structures that complement internal representations" (Barsalou, 2010, p. 716). In the context of an embodied approach to learning environments, grounding cognition involves contextualizing and situating action within the goal structures outlined. With regard to the study we conducted on grounded embodied learning environments for computational and mathematical thinking in video game design, the scenarios that define the situations and environment are critical and, as we found from the results, essential to representing cognition in embodied instruction.

For the content we used local sports teams, popular musical artists, playing video games, and homework completion topics to ground the instruction in contexts familiar with our target population of suburban middle school students from the northeast. The basic structure of the pre-defined scripts used to ground the embodied actions are shown in Figure 7. Each script was match for length, structure, and sequence. In particular, following an 'hour-glass'-like shape, every scenario began with movement and a socially-guided prompt in the form of a question. Next, the learners, reading the scripts in parallel as to embody a dialogue, had to evaluate a Simple or Complex Conditional Statement (Fadjo et al., 2008). Based on the outcome, the sequence would continue with more dialogue and conclude with movement synonymous with the termination of a typical conversation. This grounded embodied learning environment characterizes the foundation upon which the mathematical and computational thinking concepts were taught and reinforced.

Within this comprehensive three-week curricular intervention at a suburban public middle school in New Hampshire, we compared numerous measures and outcomes from the artifacts students created and the surveys they completed to evaluate what effect embodiment and Imaginary World Construction (IWC) had on their computational and mathematical thinking. We had a Direct Embodiment with Explicit Imagined Embodiment condition (DE-EI), a Non-Physical Embodiment with Explicit Imagined Embodiment condition (X-EI), a Continuous Imaginary World Construction (IWC-C), and a Discrete Imaginary World Construction (IWC-D) condition. In the Imaginary World Construction conditions the students either continued the scenario previously defined during the Code Literacy instructional session (IWC-C) or developed a completely new scenario (IWC-D) within the same constraints offered to the continuous group.

With the grounded embodied conditions, we found that not only did those who engaged in Direct Embodiment and Explicit Imagined Embodiment (DE-EI) during instruction utilize significantly more mathematical structures in their artifacts, but that those who engaged in DE-EI also wrote significantly more code structures within their video game/story artifact. Thus, the mere fact that students physically enacted pre-defined code structures for five minutes at the beginning of class resulted in artifacts that were mathematically more complex, had significantly more code structures (and often utilized more complex code structures), and showed more evidence of computational thinking (in particular, decomposition of pre-defined exemplars into individual artifacts). We believe that this grounded embodied learning environment design can extend beyond the language with which computational thinking is historically taught (namely, computer programming languages such as *Scratch*, the block-based programming language used in this experiment, Resnick, 2009) to other domains and topics such as word problems, geometric patterning, or probability thinking where abstract concepts are fundamentally challenging for the instructor to teach and the student to learn in formal classroom settings.

In the Imaginary Worlds Construction conditions, we found a strong correlation between the option to construct an Imaginary World distinctly different than the initial video game world offered to all students (IWC-D) and the self-reported satisfaction of doing the task. Coupling the ability to construct an Imaginary World that is individually meaningful with a grounded embodied approach to learning abstract computational and mathematical concepts is evidence that a Constructionist (Papert, 1980, 1991; Harel & Papert, 1990; Harel, 1991; Harel Caperton, 2010) learning environment where grounded (Barsalou, 2008, 2010) embodied cognition (Glenberg et al., 2004, 2009) is coupled with Imaginary World Construction (Fadjo & Black, 2011) that is constrained, but individually meaningful leads to significant gains in mathematical and computational thinking.

Insert Figures 6 and 7 about here.

Similarly, Lu, Black, Kang and Huang (2011) found that using direct physical embodiment (having students act out with their own bodies before programming) together with robot programming surrogate embodiment led to the best learning of physics concepts. In this study, having the students embody their understanding of physical science concepts (force and motion) by building and programming robot surrogates (using LEGO Mindstorms NXT robots) increased their understanding and learning. Further, having the students directly physically embody the physics concepts by acting out the situations first with their own bodies and then programming their robots to do the same thing left to much greater learning and understanding. Thus, having students directly experience something then imagine these experiences and embody their understanding in robot surrogates lead to the best learning and understanding of all. This combination (experience, imagination, surrogate embodiment) also led to large increases in students interest in physics topics and their confidence that they had understood them.

Conclusions

We have provided a variety of examples of how one can design learning environments using an embodied or perceptually-grounded cognition approach, and that this kind of design can lead to greater student learning, understanding and performance in addition to increasing student interest in what they are learning and confidence that they have mastered it. Specifically, in the studies summarized here we have a number of results that yield more effective learning, understanding and motivation when designing embodied learning environments:

1) The richer the perceptual environment using multiple sensory modalities (e.g., using visuals, voiceovers, and movement) during initial learning the better the student learning, understanding and motivation
2) Utilizing movements (e.g., gestures) that are conceptually congruent with the knowledge being learned increases student performance, learning, understanding and motivation

3) Having students directly experience a phenomenon through activities like acting it out moving their own bodies, then learning about it in a more general way increases student learning, understanding and motivation
4) Having students embody their understanding in surrogates then observing the surrogate behavior through activities like programming video-game-like virtual environments with avatar surrogates and programming robot surrogates like the LEGO NXT ones, increases student learning, understanding and motivation.

Recent inexpensive technology developments provide tools that make implementing theses embodied learning environments easier: e.g., touch-gesture interfaces like the iPhone and iPad, simple programming tools like the *Scratch* programming environment and simple robot kits and programming like the NXT LEGO robots. We think that this approach provides a way to design learning environments that produce learning that becomes more a part of the way students think about an interact with the world, and thus will lead to greater transfer of learning beyond classroom settings. Fortunately, current technology developments are providing more and better ways to produce these embodied learning environments integrated with the real world. For example, our group is currently working on a new generation of embodied computer simulations using the Microsoft *Kinect* general gestural and speech interface.

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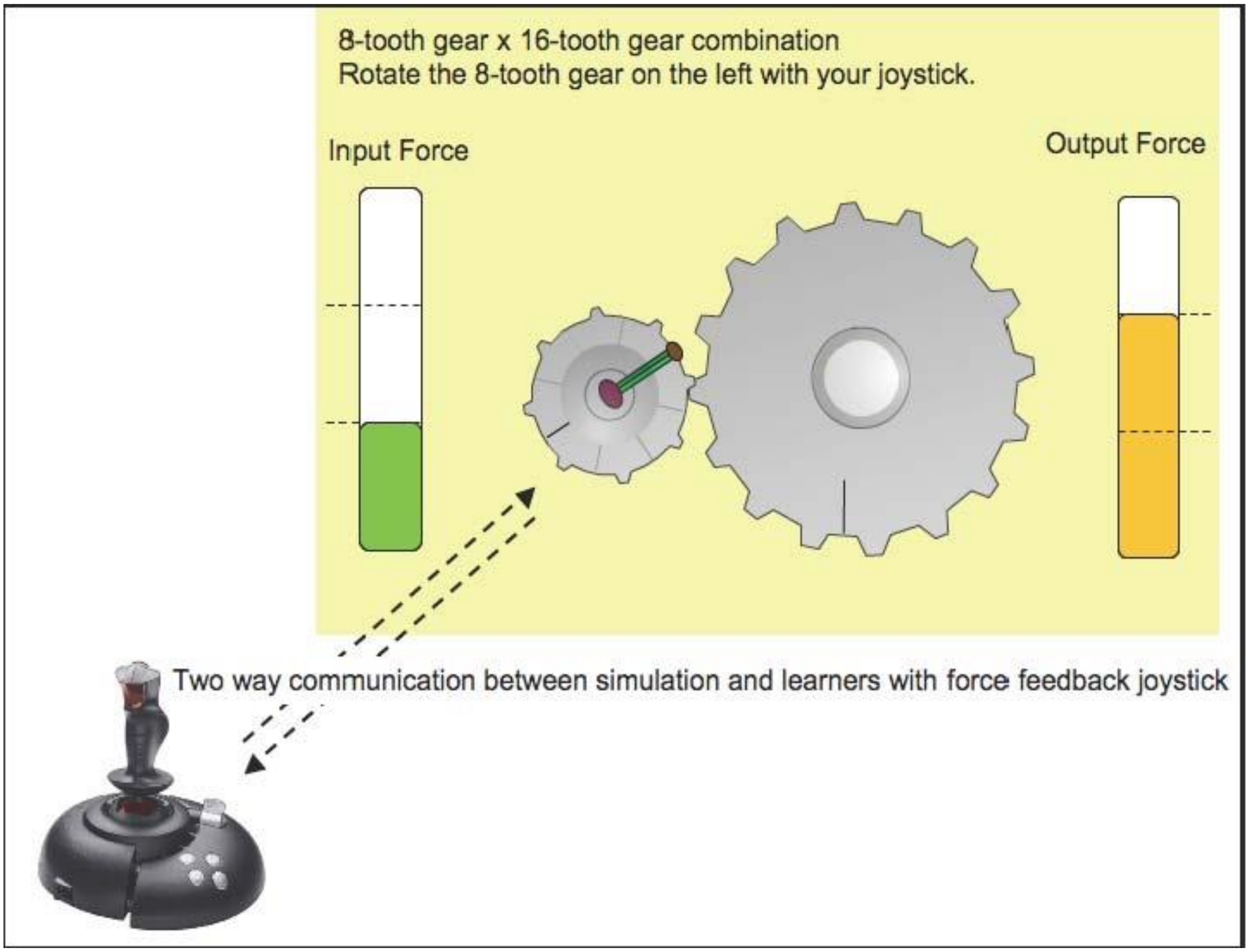

Figure 1 Gear Graphic Simulation with Movement (Joystick and Gears) and Animation with Force Feedback (Joystick)

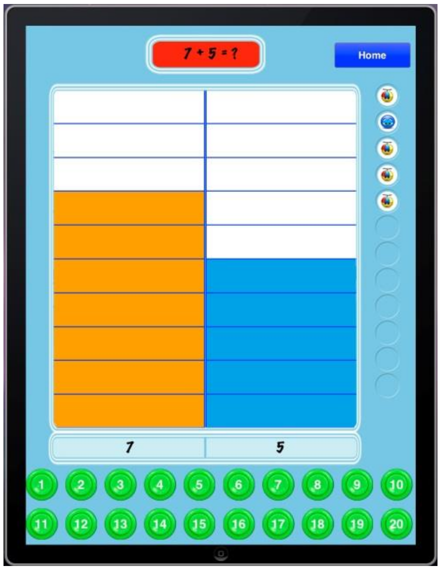

Figure 2. Counting and Addition task interface

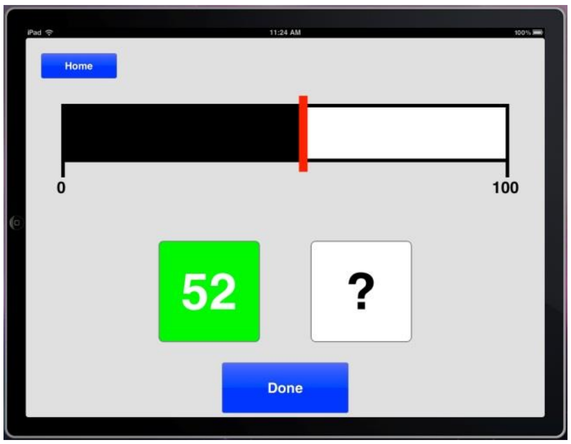

Figure 3. Number line estimation task interface

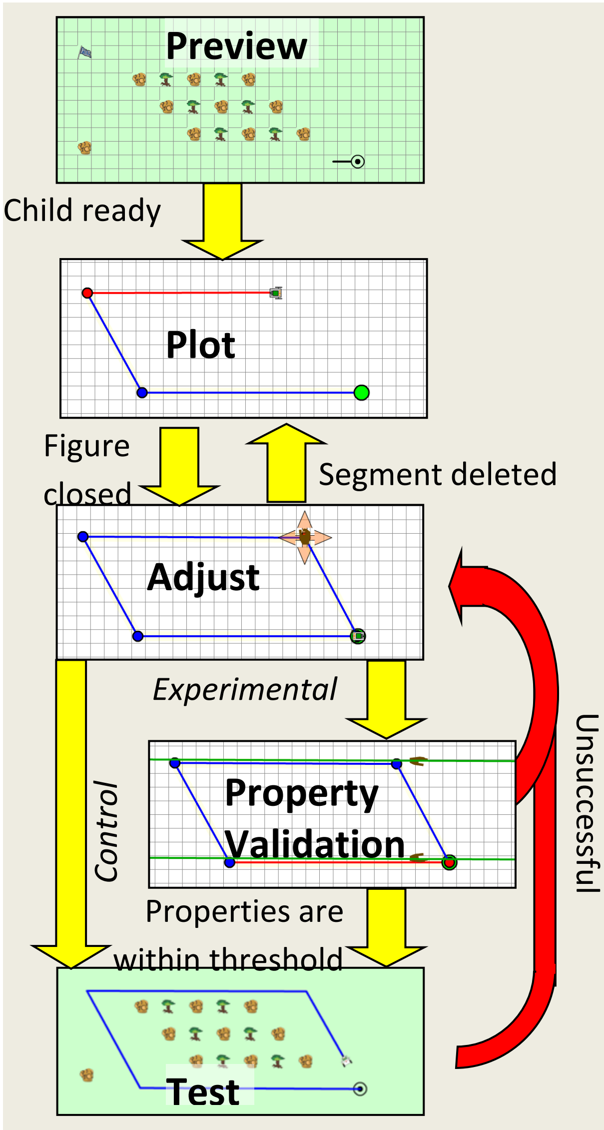

Figure 4. Polygon construction game flow, with cropped screenshots from a parallelogram task. Yellow arrows demonstrate progress in the intended direction. Red arrows demonstrate mistake-based reversals.

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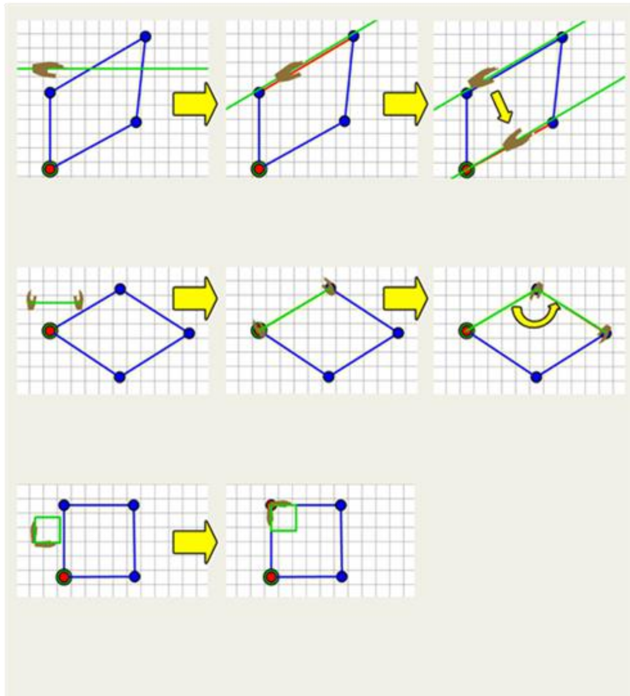

Figure 5. Visual depictions in property validation phase. Displays validation of parallel sides, congruent sides, and right angles (from top to bottom).

Figure 6. Instructional Embodiment Framework (IEF)

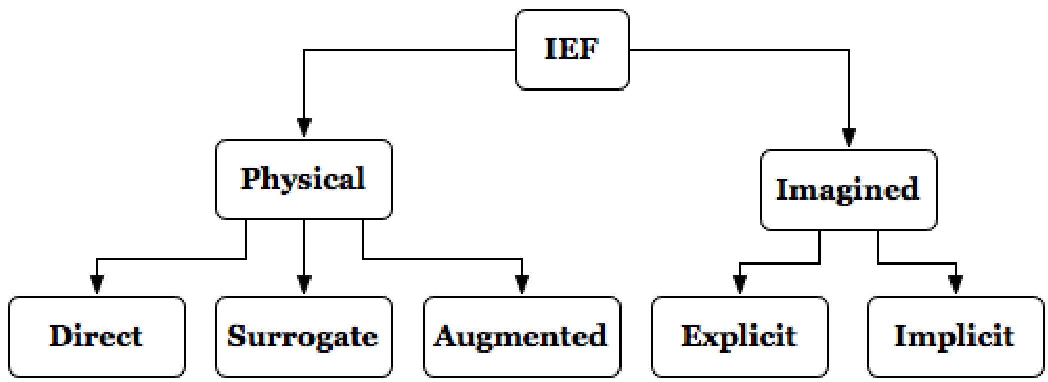

Figure 7. Embodiment Scripts