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Technical Report MES-06-01:

Revised Coding Manual for Identifying Item Involvement of Content, Context, and Process Subskills for the TIMSS and TIMSS-R 8th Grade Mathematics Tests

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SUMMARY

A set of proposed cognitive “attributes” or subskills involved in the solution of mathematics test items is described. These attributes correspond to specific aspects of 1) mathematics content knowledge, 2) cognitive processing abilities, and 3) test-taking and problem-solving skills believed to be required for successful solution of TIMSS mathematics items (and similar tests). The proposed attributes, which we refer to as the “TCD” attributes to distinguish them from facets of the test framework used by TIMSS-R test developers, were developed specifically for analyzing mathematics achievement on the TIMSS-R (1999) 8^{th} grade achievement data, and for the TIMSS (1995) 8^{th} grade or 12^{th} grade general math test. Attributes for the TIMSS 1995 12^{th}-grade Advanced Math test will be described in a subsequent report. The TCD attributes may be viewed as an extension and refinement of the TIMSS-R (1999) test framework, which was based on a curriculum framework entailing classification of test items by “content” and “performance expectations” (Robitaille, D. F., Smith, W. H., Raizen, S., McKnight, C. Britton, F. E., & Nicol, C., 1993). However, in contrast to that TIMSS-R (1999) mathematics framework, in which the different classes of the content dimension and of the performance dimension were mutually exclusive, the current content, process, and skill attributes are not assumed to be mutually exclusive.

This Technical Report is in four parts. Part I contains some pertinent background on the TIMSS (1995) and TIMSS-R (1999) tests. Part II is a brief description of the proposed “TCD” attributes. Part III is a more detailed description of the definitions of attributes and the coding rules. Part IV contains coded examples of TIMSS-R math test items.

CONTENTS

I. Background and Method

The TIMSS (1995) and TIMSS-R (1999) tests

The Third International Math and Science Study, or TIMSS (1995) was an ambitious project designed to measure educational achievement in math and science, across grade levels from 4 to 12 and in more than 50 countries. The follow-up TIMSS-R (1999) study focused on Grade 8 math and science achievement, and gathered more detailed data on curriculum, school, teacher, and student background variables, plus benchmarking data from 27 jurisdictions in the U.S. Thirty-eight countries participated in the TIMSS-R (Mullis, Martin, Gonzales, Gregory, Garden, O'Connor, Chrostowski, & Smith, 2000). The data collected is many-faceted, including not only achievement data from the actual math and science test, but also background questionnaires aimed at measuring various aspects of students, teachers and schools, and even including videotaped observations of actual math and science lessons in the various countries.

The TIMSS-R (1999) Test Framework

The TIMSS-R test items were classified according to a framework that assigned each item to one of five content areas and one of five "performance expectation" classes. The five content areas of the test were: 1) Fractions and number sense, 2) Measurement, 3) Data representation, analysis, and probability, 4) Geometry, and 5) Algebra. The five performance areas were: 1) Knowing, 2) Using routine procedures, 3) Using complex procedures, 4) Investigating and solving problems, and 5) communicating and reasoning. Each item was classified into a single content class and a single performance class. In addition, three different item formats were used: multiple choice, short-answer, and extended response.

Development of Attributes for the Present Study

For the present study, we identified attributes believed to be involved in successful solution of the TIMSS mathematics items by the following method. First, we asked a team of domain experts (faculty and graduate students in measurement and statistics, including several former high school mathematics teachers) to solve the complete pool (163 items) of TIMSS-R (1999) "population 2" (eighth grade) mathematics test items. These experts individually generated proposed attributes corresponding to important facets of content knowledge, and process subskills. As an additional source of information on the processes by which students solve these mathematics items, we also collected and analyzed written protocols from four high school students who were asked to solve all the items. The attributes generated by this process were then jointly discussed, combined and refined. The attributes were classified as being one of three general types. The first group of attributes consists of content knowledge attributes (see Section II), that is, clusters of content knowledge. The second group consists of attributes describing special-purpose test-taking skills, some associated with a particular type of test item (for example, items with "open-ended" answer formats). This group is called "skill" or item type" attributes. Finally, the third group of attributes describes individual process subskills involved in solving the test items. These attributes are listed in Section II, and described more fully in Section III. Procedures used to validate the attributes and validation results are discussed in Corter and Tatsuoka (2002).

Some Theoretical and Practical Issues:

In summarizing various coding issues, it is probably worthwhile to begin by re-emphasizing that the current content, process, and skill attributes are not assumed to be mutually exclusive. This is in contrast to the TIMSS (1995) and TIMSS-R (1999) mathematics frameworks, in which the different classes within the "content dimension" (Geometry, Algebra, etc.) and the classes of the "performance dimension" were mutually exclusive, so that an item could not be classified as having both Geometry and Algebra content. In our scheme, an item might tap content knowledge in both of these areas, though it is probably true that test designers generally attempt to avoid writing items that tap multiple content or multiple process skills.

1) *Multiple solution methods/strategies.* One complication that arises in coding an item for attribute involvement (that is, in specifying the content, skill, and process attributes that are required to solve it) is the fact that different students may use different methods to solve the same item. In fact, it is possible (though perhaps rare on a speeded test) that an individual student might use more than one approach on a single item. Another issue is that the method used by a coder who is an expert in mathematics may differ from the approach typically used by students in a particular population (e.g., 8^{th} graders). This problem calls for caution in coding, and suggests that any coding project ought to involve the collection of "show-your-work" written protocols to help ensure that the coded skills are actually the ones used by a majority of students. This potential problem leads to the following recommendation.

2) *Code strategies, not items.* When a single mathematics item may be solved by more than one possible solution method, it is probably best to separately code *each* possible solution method for its required component subskills (attributes). In the final Q matrix, the researcher may wish to use the coding for the solution method that seems to be employed by the largest subgroup of students. The issue of which method is used by the largest number of students can be established empirically, by collecting protocol data, or an attempt can be made to establish this statistically. An example of the latter approach is that of Tatsuoka (1983), who attempted to establish the dominant method used by a population of students for mixed-fraction subtraction by coding attributes for both of the two most common methods, then testing which version of the Q matrix best predicts item difficulty for the group data. An alternative approach is to code the skills and knowledge required to solve the method by the more standard or formal method, if that can be established by expert judgment.

Examples of items that admit of multiple solution methods are Items B08 and R15 in the TIMSS-R (1999) Population 2 test, which can be solved either by algebra or by simpler methods (see Section IV below). It should be noted that for large-scale highstakes tests such as the SAT, test prep instruction often teaches “non-mathematical” strategies for finding the correct alternative in multiple-choice tests.

3) *Setting thresholds for attribute involvement in an item.* There are several issues affecting how attribute involvement in an item is defined, and decisions on these issues affect the resulting item coding. First, if only trivial levels of a content or process subskill are needed to solve the item (by a particular method), that attribute is not coded. Of course, defining what is a trivial level requires setting a somewhat arbitrary threshold. This can be a source of disagreement among raters. Thus, some discussion and joint coding of items will generally be necessary to calibrate multiple raters. The above issue arises more with certain attributes than with others. For example, it is usually clear whether or not an item involves geometry content knowledge (attribute C4), but many items require some degree of integer arithmetic skills (C1), requiring a decision on setting a threshold in order to code involvement of C1 reliably. In our coding we adopted specific rules about what level of integer operations skills exceeded threshold. For example, we did not code C1 in the case of the simple operations addition, subtraction, multiplication, or division of two single-digit numbers (for an 8^{th} grade population). In the absence of such specific rules, varying thresholds between raters is one of the two most important sources of rater disagreement (along with use of different solution strategies for an item).

4) “Component coding” versus “Focus coding” of content knowledge. The published test design frameworks for the TIMSS and TIMSS-R tests specify that the Content categories of items are defined to be mutually exclusive. For example, an item is described in the TIMSS test design framework as dealing with Geometry or Algebra, but not both. We refer to this type of coding as *focus coding* of Content, because enforcing the requirement of mutual exclusivity requires that the coder make a decision as to the *primary* content (or the primary purpose of the item from an educational measurement perspective).

In contrast, a complete specification of the cognitive and mathematical skills required in solving an item would requires that *all* necessary content knowledge be recognized in the coding (*component coding* of Content), as long as the requirement for that category of skill is non-trivial. Consider item B08:

> “If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?”

A student who chose to solve this problem *algebraically* might set up this problem as an equation with one unknown:

$$300 /100 = x / 30,$$

then rearrange it to solve:

$$x = (300)(30)/100 = 90.$$

Under focus coding, such an algebraic solution for this item might be coded as focusing on attribute C3 (Algebra). However, under component coding, we might code C1 and C2 in addition to C3, because we recognize that integer arithmetic skills (C1) and fraction/ratio manipulation skills (C2) are required for simplifying the ratio expression to find the exact answer.

Depending on the researchers’ purposes, one or the other of these coding conventions might be followed. However, componential coding of content seems more in line with prominent theories underlying diagnostic testing. In general, we recommend component coding of content and have used that type of coding in the majority of our reanalyses of student performance in TIMSS and TIMSS-R.

II. List of the proposed TCD attributes for TIMSS-R mathematics

A list of knowledge, skill, and process attributes to explain performance on the TIMSS-R (1999) math items, and 1995-general mathematics in Population 2 (8^{th} graders).

CONTENT ATTRIBUTES

C1 Basic concepts and operations in whole numbers and integers
C2 Basic concepts and operations in fractions, decimals, and ratios
C3 Basic concepts and operations in elementary algebra
C4 Basic concepts and operations in geometry
C5 Data techniques; probability and basic statistics
C6 Measurement and estimation

PROCESS ATTRIBUTES

P1 Translate words by formulating algebraic equations or algebraic expressions
P2 Computational applications of knowledge in arithmetic, algebra and geometry
P3 Judgmental applications of knowledge in arithmetic, algebra and geometry
P4 Applying complex rules in algebra
P5 Logical reasoning: case reasoning, induction and deduction, necessary and sufficient conditions, generalization skills
P6 Problem-space search: analytic thinking; problem restructuring; search for novel strategies
P7 Generating, visualizing or transforming figures and graphs
P8 Recognizing and evaluating mathematical correctness; proof skills
P9 Managing complex data, procedures, subgoals or conditions
P10 Reading and understanding comparative or logical terms

SKILL AND ITEM TYPE ATTRIBUTES

S1 Unit conversion
S2 Apply number properties and relationships; number sense/number line
S3 Using provided figures, tables, charts and graphs
S4 Approximation/estimation
S5 Evaluating/verifying/checking solutions
S6 Recognizing serial patterns
S7 Proportional reasoning
S10 Open-ended items
S11 Reading/understanding problems with complex text or real-world context

III. Detailed Descriptions of the TCD Attributes

Content Attributes

C1: Basic concepts and operations in whole numbers and integers
C2: Basic concepts and operations in fractions, decimals, and ratios
C3: Basic concepts and operations in elementary algebra
C4: Basic concepts and properties in geometry
C5: Data techniques; probability and basic statistics
C6 Measurement and estimation

C_1 (Whole Numbers and Integers)

A student who has mastered this attribute should be able to understand and apply basic concepts and operations in whole numbers and integers, including addition, subtraction, multiplication, division, exponentiation, sign, absolute value, place value, etc. The attribute does not include simple comparisons of the magnitudes of integers or digits (which of two numbers is larger or smaller). It does include rounding of 2- or 3-digit integer values.

C_2 (Fractions, Decimals, Ratios)

A student who has mastered this attribute should be able to understand and apply basic concepts and operations in fractions and decimals, such as setting up ratios, using them in calculations and equations, use of mixed fractions, decimals, etc. Note that setting up ratios of integers and evaluating them could be considered to be merely a basic operation involving integers (C1), but we argue that the skills involved are somewhat distinct and more closely allied with other skills in C2, thus should be coded as C2. Rounding and ordering of decimal quantities and fractions is included.

C_3 (Algebra)

A student who has mastered this attribute should be able to understand and apply basic concepts and operations in elementary algebra, such as representing an unknown by a symbol, simplifying expressions and solving equations involving an unknown, etc.

C_4 (Geometry)

A student who has mastered this attribute should be able to understand and apply basic concepts and operations in geometry, such as knowing basic geometric concepts, shapes, property of polygons, Cartesian coordinates, etc. The concept of distance as applied to a 1-dimensional space (number line) is included, but not the use of a number line to represent simple magnitudes.

C_5 (Data, Probability, and Statistics)

A student who has mastered this attribute should be able to interpret and manipulate data in simple ways, and to use data already represented in graphs, charts, or tables. Also, the student should understand basic probability concepts, and be able to compute and interpret basic statistics (for example, understand the concept of the mean and calculate it).

C_6 (Measurement and Estimation)

A student who has mastered this attribute should be able to measure (or estimate) length, time, angle, coordinates, temperature, etc., using appropriate tools and appropriate units. The knowledge necessary to convert units of measurement is also included (e.g., the number of milliliters in a liter; the number of seconds in a minute, etc.)

Simple rounding of numbers and numeric place value does not count as this attribute (knowledge of place value would be C1 or C2), nor does simple visual estimation.

Process Attributes

P1 Translate words by formulating algebraic equations or algebraic expressions
P2 Computational applications of knowledge in arithmetic, algebra and geometry
P3 Judgmental applications of knowledge in arithmetic, algebra and geometry
P4 Applying complex rules in algebra
P5 Logical reasoning (including case reasoning, deductive thinking skills, necessary and sufficient, generalization skills)
P6 Problem-space search: analytic thinking; problem restructuring; search for novel strategies
P7 Generating, visualizing or transforming figures and graphs
P8 Recognizing and evaluating mathematical correctness; Proof skills
P9 Managing complex data, procedures, subgoals or conditions
P10 Reading and understanding comparative or logical terms

P_1 (Translate Words by Formulating Algebraic Equations or Algebraic Expressions):

A student who has mastered this attribute (at the 8^{th} grade level) should be able to translate a word problem into an algebraic expression or equation where the designation of the variable term and constant(s) is not necessary apparent, there are no more than two unknowns, and the language in the word problem that describes the appropriate operations (e.g., subtraction) is simple (e.g., "less than"). Translation is sometimes straightforward, but most of the time it requires identification of variables and relationships buried implicitly in verbal expressions. This attribute usually refers to descriptions of “real world” problems and experiences, rather than simple verbal expression of mathematical content. This attribute does not include using given geometrical figures or spatial information.

P_2 (Computational Applications of Knowledge in Arithmetic, Algebra and Geometry):

A student who has mastered this attribute should be able to successfully *apply* knowledge about the basic terminologies, concepts and properties in arithmetic, geometry and analytic geometry and algebra. The student should to be able to perform simple computations using basic arithmetic operations, on integers, decimals, fractions, and signed numbers. This attribute (at the 8th grade level) includes such skills as interpretation of large numbers, equivalent fractions, and borrowing in mixed fractions.

The student should exhibit competence by being able to: perform basic arithmetic operations, including such skills as using a calculator, division of two fractions, taking powers and roots; perform basic operations and calculations in geometry and trigonometry; including computing volumes, areas and perimeters; etc.

P_3 (Judgmental Applications of Knowledge in Arithmetic, Algebra, and Geometry):

A student who has mastered this attribute should be able to apply knowledge of arithmetic, algebra and geometry in an abstract way to answer *conceptual* questions, or to find true relationships, properties and/or new goals in solving a problem. For example, use knowledge in arithmetic, algebra and geometry to verify the truth of statements or propositions, find the right relationships, judge the quality of answers, etc.

This attribute does not include "judgment" in the sense of estimating quantities (which would be S4).

P_4 (**Applying Complex Rules in Algebra**)**:**

A student who has mastered this attribute should be able to demonstrate proper application (computational or judgmental) of more complex algebra rules and algorithms (relative to 8th grade skills) that are clearly called for by the task. The student should be able to: solve equations; manipulate algebraic expressions; compute algebraic expressions; factor algebraic equations; factor algebraic expressions; substitute numbers or/and variables in algebraic expressions and algebraic equations; plot equations of lines, and solve equations graphically.

Coding of this attribute would normally imply that either P2 or P3 should also be coded.

P_5 (Logical Reasoning: Case Reasoning, Induction and Deduction, Necessary and Sufficient Conditions, Generalization Skills):

A student who has mastered this attribute should be able to reason from cause to effect, and solve problems by using case reasoning to establish general principles. However, this attribute is not necessarily coded when the cases are given explicitly in the multiple-choice options or otherwise in the statement of the problem. A prototypical case of this attribute is when the cases must be chosen to cover certain critical cases, e.g. when $x>0$, $x<0$, $x=0$. This attribute implies that the student can understand necessary and sufficient conditions, and can use them to set up and conduct appropriate problem solving activities. Other implied skills include: find a counter example; check that a solution holds true in the general case as opposed to simply a specific case (i.e., check necessary and sufficient conditions); understand the conditions for solving systems or problems.

P_6 (Problem-Space Search: Analytic thinking, Problem Restructuring, Search for Novel Strategies):

The student should able to analyze the problem into component parts or constituent elements which are not obvious, or are latent or implicit; structure the component parts cognitively so that a problem become solvable; synthesize two or more concepts and theorems into a solvable form.

A student who has mastered this attribute should be able to reject a "wrong structure" which is already given, and find a new one, where the new structure is in a more solvable form. When two or more strategies exist, the student can choose the better, simpler, or quicker strategy. When two or more rules, properties, or theorems are available, the student can choose a better, simpler, or quicker one.

P_7 (Generating, Visualizing or Transforming Figures and Graphs):

A student who has mastered this attribute should be able to generate or manipulate (in explicit written form, or mentally) diagrams and figures to facilitate problem solving activities.

P_8 (Recognizing and Evaluating Mathematical Correctness; Proof Skills):

A student who has mastered this attribute should be able to work backward from the options in order to reach a solution. Such a student may solve a task by: working backwards from the multiple-choice options, producing one or two examples and from these inferring the correct option; solving the problem "intuitively" in which case the reasoning may be unconventional; substituting numbers to select the correct option; eliminating one or more of the options and guessing from among the remaining options.

P_9 (Managing Complex Data, Procedures, Subgoals or Conditions):

A student who has mastered this attribute should be able to: deal with problems that involve two or more steps/subgoals by paying explicit attention to record keeping, management of information or data, or management of a complicated chain of reasoning. Also, be able to do calculations involving management of intermediate results.

These steps can include explicit or implicit subgoals, and are not necessarily latent. In other words, the student should be able to: establish the subgoals of the problem; order and prioritize the subgoals; execute the subgoals in the proper order in a step-by-step fashion.

A student who has mastered this attribute should be able to keep track of what the question is asking, pay attention to detail, ignore irrelevant information, and identify constraints and use them in problem solving activities.

P_{10} (Quantitative and Logical Reading):

A student who has mastered this attribute should be able to: read complex, long sentences, and/or sentences with negation; use of terms such as “at least”, “at most”, “equivalent”, “must be”, “could be”, etc.; and should be able to deal with relations of increasing and decreasing quantities and comparisons. Also, understand and use logical quantifiers like “for every”, “for any”, “for a given” or “there exists”.

Note that relatively easy and everyday terms (for 8th graders) should be viewed as trivial and not coded, for example: “more”, “smallest”, “greater than”, ”equal”, “add”, “subtract”, “multiply”. Also, terms that are specialized to one particular content area, such as “isosceles” (geometry), “fair” (probability), “rounded to the nearest..” (measurement), would not be coded, because the terminology is considered inherent in the content knowledge.

<u>Skill/Item Type Attributes</u>

S1 Unit conversion
S2 Apply number properties and relationships; number sense/number line
S3 Using provided figures, tables, charts and graphs
S4 Approximation/estimation
S5 Evaluating/verifying/checking solutions
S6 Recognizing serial patterns
S7 Proportional reasoning
S10 Open-ended items
S11 Reading/understanding problems with complex text or real-world context

S_1 (Unit Conversion):

A student who has mastered this attribute should be able to convert units to another scale or unit, and compare quantities expressed in different units.

S_2 (Apply Number Properties and Relationships/Number Sense):

This attribute is coded in the presence of two somewhat distinct subskills. The first subskill involves procedural manipulations and reasoning using knowledge of formal number properties (e.g. that a negative number squared gives a positive result).

The second subskill involves using “number sense”; i.e., make experience-based judgments about relationships among numbers and quantities. The prototypical skill here is reasoning about numbers, comparing magnitudes, and other skills that require use of a mental “number line”, or equivalent reasoning (including working with an explicitly given number line).

S_3 (Using Provided Figures, Tables, Charts and Graphs)

A student who has mastered this attribute should be able to: follow written instructions that involve tables, figures, and graphs; comprehend the information contained in tables, figures and graphs, and use such information in answering questions.

S_4 (Approximation/Estimation):

These items require the students to solve problems by estimating or approximating numeric values (involving integers, reals, or fractions), or approximating quantities (including areas, and volumes in geometrical shapes) either visually or mathematically. This skill includes rounding of numerical quantities.

S_5 (Evaluating/Verifying/Checking Solutions):

For multiple-choice items, this item-type attribute is coded only when: 1) the stem of a question explicitly or implicitly (by use of phrases such as "best answer") asks the problem solver to consider or work with ALL the answer options given with an item, OR when 2) the answer cannot be generated by the student in a specific form, and therefore must be selected from the provided options.

For items that are not multiple-choice, this attribute is coded when the item requires that the student be able to verify the correctness of, or evaluate the reasonableness of, a given solution to a problem; discuss and/or critically evaluate a mathematical idea, conjecture, problem solving strategy, method, proof, etc.; provide evidence for the validity of an action or truth of a statement by reference to mathematical results or properties; or develop mathematical arguments to prove or disprove statements.

S_6 (Recognizing Serial Patterns):

A student who has mastered this attribute should be able to recognize serial patterns which may be numeric, geometric, or algebraic; find the rules of generating patterns; generate further terms in a series such as 20th or 50th terms; extend an iterative geometric pattern or diagram so that certain calculations or constructions can be made.

S_7 (Proportional Reasoning):

A student who has mastered this attribute should be able to reason about proportional relationships such as percentages and ratios, and use such relationships to solve problems. Any problem that involves reasoning about fractions or ratios is coded as involving this attribute.

S_{10} (Open-Ended Items):

A student who has mastered this attribute should be able to derive and present answers in open-ended item formats.

S_{11} (Reading/Understanding Problems with Complex Text or Real-world Context):

A student who has mastered this attribute should be able to: read and comprehend a problem text (or follow written instruction) that 1) requires real world knowledge or context for understanding, or 2) is particularly complex, or especially long. Better reliability can be obtained if an explicit cutoff is set for coding what constitutes a "long" item text (e.g., more than two lines of text).

IV. Examples with Coding and Explanations

In this section, several of the publicly released TIMSS-R (1999) 8^{th}-grade ("population 2") math items are displayed, along with their coding in terms of the TCD attributes described in this manual. The approximate difficulty of each item is (inversely) indicated by its observed mean proportion correct in the international sample (reported in parentheses). Brief explanations are also given as to why each attribute was coded for an item.

Example 1

Item T01. A club has 86 members, and there are 14 more girls than boys. How many boys and how many girls are members of the club?
Answer: _______________

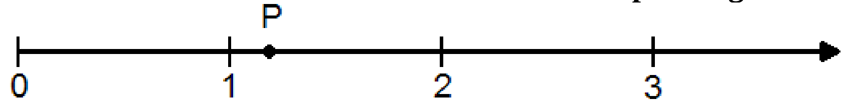
Attributes involved: C1, C3, S10, S11, P1, P2, P4, P9, P10 **(p=.33)**

Question and answer involve integer values, operations ------------------------> **C1**
Most common strategy is to set up an equation involving x --------------------> **C3**
Item type is open-ended (short answer) ---> **S10**
Traditional word problem (w/ moderate complexity) ----------------------------> **S11**
Algebraic equation must be developed to represent situation -------------------> **P1**
Computations on integer expressions are needed to solve expressions --------> **P2**
Must solve equation for x ---> **P4**
Need subgoal to represent both quantities with a single variable, or use
two equations and two unknowns --> **P9**
Quantitative reading: phrases such as "more", "less", etc. ----------------------> **P10**

Example 2

Item D12.

What is the best estimate of the number corresponding to P?



A. 1.1
B. 1.2
C. 1.4
D. 1.5

Attributes involved: C2, S2, S3, S4, S5 **(p= .68)**

Explanation for coding:
Answers are real numbers expressed as mixed decimals ---------------------------> **C2**
Visual understanding of real number line is necessary ------------------------------> **S2**
Problem is presented via a graph of the real number line ---------------------------> **S3**
Problem explicitly asks for estimation of a real quantity ----------------------------> **S4**
Examinee is asked to examine multiple options to pick the best one --------------> **S5**

Example 3

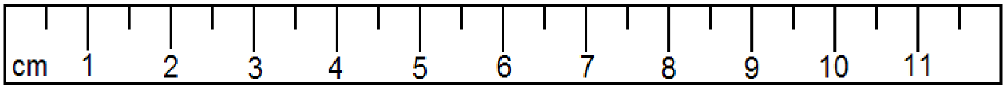
Item J11. Of the following, which is NOT true for all rectangles?
- **A. The opposite sides are parallel.**
- **B. The opposite sides are equal.**
- **C. All angles are right angles.**
- **D. The diagonals are equal.**
- **E. The diagonals are perpendicular.**

Attributes involved: C4, S5, P3, P10 **(p= .54)**

Explanation for coding:
Question requires knowledge of geometry concepts -------------------------------> **C4**
Finding best answer requires checking all options ----------------------------------> **S5**
Judgment is required to find which statements are *generally* true -----------------> **P3**
Negation in statement of problem adds complexity --------------------------------> **P10**

Example 4

Item F10.



Using a centimeter ruler like this one, you can measure accurately to the nearest
- **A. millimeter**
- **B. half-millimeter**
- **C. centimeter**
- **D. half-centimeter**

Attributes involved: C6, S2, S3 **(p= .49)**

Explanation for coding:
Question requires knowledge of measurement tools ------------------------------> **C6**
Real number line is invoked implicitly ---> **S2**
Must reason using the supplied figure (diagram of a ruler) -----------------------> **S3**

Example 5

Item L15. The table shows some values of x and y, where x is proportional to y.

X	**4**	**8**	**Q**
Y	**9**	**P**	**45**

What are the values of P and Q ?

A. **P=40 and Q=13**
B. **P=18 and Q=17**
C. **P=20 and Q=18**
D. **P=40 and Q=18**
E. **P=18 and Q=20**

Attributes involved: C3, S3, S6, S7, P2, P5, P9 **(p=.33)**

Explanation for coding:
The problem is posed in terms of variables (x and y) ----------------------------> **C3**
Problem uses a chart to present information ---------------------------------------> **S3**
The chart is used to define and present a pattern over time or cases -----------> **S6**
Since x and y are proportional, 4/9 = 8/P and 4/9 = Q/45 -----------------------> **S7**
To get the values of P and Q, need to compute quantities or solve ------------> **P2**
Setting these equations up is not straightforward because P is in numerator
but Q is in denominator: must consider generality of solution method -----> **P5**
Two subparts to problem, therefore it is multi-stage ----------------------------> **P9**

EXAMPLES WITH MULTIPLE STRATEGIES:

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Example 6

Item R15. John sold 60 magazines and Mark sold 80 magazines. The magazines were all sold for the same price. The total amount of money received for the magazines was $700. How much money did Mark receive?
Answer: _______________

Strategy A: ARITHMETIC SOLUTION METHOD
Attributes involved: C1, S10, S11, P1, P2, P9 (p=.44)

Explanation for coding:
Stated quantities in problem and goal are integers --------------------------------> C1
Open-ended item ---> S10
Problem cover story involves real-world story, fairly complex ------------------> S11
Solver must formulate math expressions to solve ----------------------------------> P1
Solver must evaluate expressions and do basic computations to solve ---------> P2
Problem has two subgoals: find cost per magazine, then find Mark's amt -----> P9

Strategy B: ALGEBRAIC SOLUTION METHOD
Attributes involved: C1, C3, S10, S11, P1, P4, P5 (p=.44)

Explanation for coding:
Stated quantities in problem and goal are integers --------------------------------> C1
Algebra: Set up 2 equations w/ 2 unknowns (= # books sold by each) ---------> C3
Open-ended item ---> S10
Problem cover story involves real-world story, fairly complex ------------------> S11
Solver must formulate equations and expressions to solve -----------------------> P1
Student must know how to solve 2 equations in 2 unknowns --------------------> P4
Some logical inference is required to turn given info into equations ------------> P5

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Example 7

Item B08. If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?

- **A. 90**
- **B. 100**
- **C. 900**
- **D. 1000**
- **E. 9000**

Strategy A: ALGEBRAIC SOLUTION METHOD (p= .69)
Attributes involved: C1, C2, C3, S7, S11, P1, P2

Explanation for coding:
Given information and expected answer are integers ------------------------------> **C1**
But solution involves setting up/solving expressions using ratios ---------------> **C2**
If student uses an explicit unknown, algebra is involved -------------------------> **C3**
Proportional reasoning is needed to set up equation correctly --------------------> **S7**
Question involves complex syntax & real-world content -------------------------> **S11**
Solution involves setting up and solving a proportional equality ----------------> **P1**
The equality must then be solved for the answer -----------------------------------> **P2**

Strategy B: PROPORTIONAL REASONING METHOD
Attributes involved: C1, C2, S7, S11, P2

Explanation for coding:
Given information and expected answer are integers ------------------------------> **C1**
But solution involves setting up expressions using ratios -------------------------> **C2**
Proportional reasoning is needed to set up ratios correctly -----------------------> **S7**
Question involves complex syntax & real-world content -------------------------> **S11**
The derived expression must then be reduced for the answer ---------------------> **P2**

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